T. P. Burgess

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NAVIGATION FOR YACHTSMEN.

NAVIGATION for YACHTSMEN.

A

CONCISE TREATISE ON NAVIGATION

AND

NAUTICAL ASTRONOMY,

ILLUSTRATING THE MOST MODERN METHODS,
AND SPECIALLY DESIGNED FOR THE
USE OF YACHTSMEN.

COMPILED BY

VINCENT JOHN ENGLISH,

RETIRED LIEUTENANT R.N.

CHEAP EDITION,

WITH ILLUSTRATIONS AND CHARTS.

London:
HORACE COX,
WINDSOR HOUSE, BREAM'S BUILDINGS, E.C.

—

1906.

LONDON:
PRINTED BY HORACE COX, WINDSOR HOUSE, BREAM'S BUILDINGS, E.C.

TO

THE RIGHT HONOURABLE

THE EARL OF DUNRAVEN, K.P.,

THIS WORK,

INTENDED FOR THE PROMOTION OF SCIENTIFIC NAVIGATION

AMONGST YACHTSMEN,

IS MOST RESPECTFULLY DEDICATED

BY THE AUTHOR.

SCHOOL OF NAVIGATION FOR YACHTSMEN,

10, *Duke Street,*
St. James's, S.W.

Vincent J. English, Retired Lieutenant, R.N., gives instruction to Yachtsmen, exclusively, in the practice and theory of Navigation and Nautical Astronomy. The theory and use of the sextant thoroughly explained. Clients are prepared to pass the Board of Trade Examinations. Lectures will be given at the above address during the winter months. Arrangements may be made for practical instruction on board yacht owners' own vessels.

For Terms, &c., apply to Lieut. English at the above address.

The highest references given.

PREFACE.

This book has been written with the idea that in dealing with men of cultivated minds it is not sufficient to lay down certain bald formulæ which require to be learnt parrot-like ; an appeal is made to their intelligence by showing, as concisely as possible, whence such formulæ are derived.

The theoretical part of the work is, however, altogether subordinated to the practical method of navigating a ship. A feature is made of the double chronometer problem, the manner of its solution being the briefest extant, without any sacrifice of accuracy.

The practice of navigation is also illustrated by the logs of two cruises, one in an iron steamer and the other in a sailing vessel. For the purpose of these cruises, and for the projection of double chronometer problems, an excellent chart of the Channel is published at the end of the book. Numerous examples are given.

The author has had the great advantage of the advice and assistance of Mr. Augustus G. Wildy, whose experience in yachting navigation is widely known.

PREFACE TO CHEAP EDITION.

In presenting this cheap edition to the public I venture to hope that the reduction in the price will, perhaps, place the book within the reach of all yachting gentlemen.

The two exercises at the end of the work, giving the actual method by which a vessel is navigated, are, in the opinion of good practical men, invaluable to the earnest student.

Vincent J. English,

April, 1906.

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NAVIGATION FOR YACHTSMEN.

CHAPTER I.

INTRODUCTION.

It may be well, by way of preface, to place before my readers the reasons which induced me to write the articles "On Navigation for Yachtsmen," originally published in the *Field*, and on which this book is founded. There are already in existence many works on navigation, some of them appealing directly to the yachting world, and others to practical seamen in the Royal Navy and Merchant Service. These latter are too voluminous, and embrace more subjects than are necessary to the ordinary yachting-man, while the former appear to me to fail in giving sufficient theory to make the practical working of the various problems intelligible. One reads in them that by adding the logarithm of the cosine of this angle to the logarithm of that number the logarithm of a certain other number required is obtained—a sort of magical mumbo-jumbo arrangement—but they do not appeal to the intellect by explaining the reason thereof. And the working out of the various problems is made to depend upon the mechanical application of certain trigonometrical formulæ, which must be committed, parrot-like, to memory, and which must be accepted in blind faith.

I do not hesitate to stigmatise this system as a bad one; and in this book I shall endeavour to supply the defect by explaining the formulæ which may be used.

It is undoubtedly a great advantage to owners of yachts to qualify themselves to command their own vessels by passing the Board of Trade examination. A practical knowledge of navigation confers a sense of authority which is necessarily absent in the man who is entirely dependent upon the judgment and skill of the master he employs, to say nothing of the inconvenience which may arise in a foreign port where the certificated master is alone recognised by the authorities. One of my objects therefore is to enable my readers to pass this examination, and so obtain a certificate from the Board of Trade, at least so far as navigation is concerned.

Navigation may be defined as the art of conducting a ship from one distant port to another. There are two methods by which the place of a vessel at sea may be determined, called respectively dead reckoning and observation. By dead reckoning is meant estimating the position of a vessel by noting down the distances and directions in which she has sailed from a known point; and by observation is meant finding out her place on the globe by astronomical calculation.

If the sea were a smooth, plane surface, without currents or tides, it would be a simple matter to fix accurately the position of a vessel, and to take her from one place to another on the earth's surface by dead reckoning only; but, as it is in constant motion, influenced by irregular currents and tides, and the drift of the waves, it becomes necessary to have some more accurate method to ensure safe navigation; and this is to be found in the system of observation of the heavenly bodies, or, in other words, in the science of nautical astronomy.

Let us consider for a moment, in a superficial manner, the reason why the waters of the ocean are always moving and never still. Suppose that a globe like the earth is covered with water over all its surface, that it also has an atmosphere, and that it is revolving, as the earth does, from west to east; but that it is all alone, without sun, moon, planets, or stars. The motion of the earth round its axis would cause no movement in the water surrounding it. Now, let us suppose that the sun commences to give its light and heat to this globe, and that it is situated perpendicularly over the equator. What would happen? The water about the equator would be heated, and evaporation would take place to such an extent that water would begin to flow from the poles towards the equator, to restore the equilibrium lost by the displacement of that evaporated. We know that a particle on the earth's surface has a greater or less velocity of rotation as it is nearer or further away from the equator. This velocity ranges from about 1000 miles an hour at the equator to nothing at the pole itself, where the particle would simply be turned round every twenty-four hours. Now, this water coming from the poles has only the slower velocity of those regions, and, in consequence, as it gets nearer to the equator. it is left behind by the faster moving surface of the earth over which it is flowing, and hence it assumes a more and more westerly direction, till at the equator, where the two polar currents meet, they together form a great equatorial current flowing round the globe from east to west. Such would be the effect if there were no obstacles such as continents, islands, &c. In the case of the earth, the water flows from the polar regions to the equator, and the great equatorial current exists ; but in the Atlantic Ocean it strikes against and is divided by the American continent into two currents, one of which runs south along the east coast of South America, and eventually dissipates itself in the South Atlantic, the other, and by far the greater portion, sweeps round the gulf of Mexico, passes Florida, and, as the well-known Gulf Stream, flows to the north-eastward across the North Atlantic, a gigantic hot salt-water river, with clearly-defined limits or banks of cold water, for a thousand miles or more, and whose waters eventually beat against the shores of North-Western Europe and the British Islands, ameliorating the climate, and rendering habitable that portion of the globe which would otherwise be another Nova Zembla.

The equatorial current in the Pacific Ocean is split up on arriving in the neighbourhood of New Guinea and the Philippine Islands, one part going north along the coast of China, another south by the East Coast of Australia, and the third and smaller one making its way through the Indian Archipelago, and continuing its course through the Indian Ocean, to be broken up and divided against the East Coast of Africa.

Only twice in each year does the sun shine perpendicularly on the equator, or, in other words, cross the line ; and these occasions are called the vernal and autumnal equinoxes. All the rest of the year he is apparently travelling either to the northward or to the southward.

This is caused by the inclination of the earth's axis to her orbit, her path round the sun during the year, which causes her to bow first one pole and then the other to this luminary. At the same time he is always comparatively near the equator, so that the water thereabouts loses more by evaporation than that in any other portion of the ocean, and consequently the effects above described are produced.

If these currents were uniform in their direction and velocity, by making due allowance for them we could, no doubt, navigate by dead reckoning alone. But they vary continually, and there are other disturbing elements to be dealt with. Our satellite the moon being so near the earth has a very powerful attraction for the waters of the earth, the effect of which is that a tidal wave, or lumping up of the waters of the ocean follows that body as the earth turns round on its axis, and another lumping up of waters takes place exactly on the opposite side of the earth to that tidal wave nearest to the moon, which is the reason why there are two high tides and two low tides every twenty-four hours. The sun in a lesser degree also attracts the waters, and this accounts for the fact that at new and full moon—that is when the sun and moon are pulling together—we have spring tides; and that at the first and third quarters of the moon, when the sun and moon are pulling at right angles to each other, we have neap tides.

Another cause of disturbance is the effect of the wind in creating temporary surface currents. It is well known that when running before the trade winds ships are nearly always ahead of their dead reckoning; that is to say, that they have sailed a greater distance over the earth's surface than they have actually gone through the water, and this because the water itself has been moving with them. But every gale of wind causes a surface current if it blows continuously in one direction for any length of time, and this current lasts often for some days after the wind has fallen. A good example of an irregular current is that which sometimes runs to the north-westward, across the mouth of the English Channel, after a heavy north-westerly gale. The cause of it is supposed to be that the water in the Bay of Biscay gets piled up by this particular wind, and when the weather moderates it recovers its equilibrium by running out past Ushant and towards the Scilly Islands. The occasional existence of this current makes good mariners, when approaching the Land's End in thick weather, give it and the Scilly Islands a wide berth. Not many years ago a splendid mail steamer was wrecked in this locality with great loss of life, having been thrown out of her course by this dangerous stream.

Another cause of error is the effect of a heavy sea, particularly on small vessels, by sagging them bodily to leeward.

And last, but not least, the uncertain errors of the compasses through the change of latitude, through alterations in the ship's sub-permanent magnetism, through the neighbourhood of land, and through many other causes, render navigation by dead reckoning alone untrustworthy.

When off a known coast in clear weather it is easy to fix the position of a ship by cross-bearings of land marks, such as lighthouses, church towers, points of land, beacons, &c., which are shown in the chart. And in the open sea, so long as the sun shines and the chronometers keep their rates, there is little anxiety in navigating a vessel, if a proper look out is kept to avoid collisions. But when one approaches a dangerous coast in thick, blowing weather, and has not had a glimpse of the sun for several days, the navigator's sheet anchor is the lead. Thousands of vessels have been lost through carelessness in this respect. Modern charts are so accurate that it is nearly always possible to verify the position of a vessel in narrow waters by taking soundings at short intervals, when the varying depths of water and the nature of the bottom, a portion of which is brought up by the arming of grease at the end of the lead, will in most cases indicate her place on the chart quite accurately enough to avoid danger. No precaution should be neglected by the navigator to make sure of his position, no matter what faith he may have in the accuracy of his compasses, the distance he has run, and his knowledge of the tides and currents. He should always remember that causes quite beyond his ken very often make the tides vary in strength and duration, and that a course which it would be quite justifiable to steer in clear weather, would become a criminally imprudent one under opposite conditions.

I will now, as briefly as possible, state the course of study presented in this work.

In order to acquire an intelligent comprehension of the formulæ to be used, a slight knowledge of algebra is requisite, and the next chapter is devoted to an explanation of the elementary rules of that science. A short chapter on decimal fractions follows. I then illustrate the theory and practice of logarithms, followed by plane trigonometry on which all plane sailing, traverse sailing, parallel sailing, mid-latitude sailing, and current sailing depend. Definitions and the construction of astronomical diagrams are then treated. Spherical trigonometry is the next subject very briefly attacked, in order that the formulæ necessary for the solution of problems in nautical astronomy may be understood. A short essay on magnetism is succeeded by an explanation of the principle of the mariner's compass, and the means employed for correcting it in iron ships. A chapter on the use of certain instruments used in navigation comes next, and then dead reckoning. After which we practically apply the work already gone through to the solution of problems in nautical astronomy, for the purpose of finding latitude and longitude by observations of the sun, moon, and stars, as also compass correction and error of chronometer. The new navigation is then thoroughly investigated and exemplified. This is followed by chart work. The use of the barometer, the law of storms, and a consideration of the tides bring us to the last chapter, which deals with the general duties of the navigator at sea.

Readers of this work who have not forgotten their mathematics will of course skip the succeeding chapters on algebra and trigonometry. They may be useful, however, to those whose memories need refreshing in those subjects.

I am strongly of opinion that the use of haversines is a distinct advantage to the student of navigation, and I recommend my readers to provide themselves with "Inman's Nautical Tables." This work is almost exclusively used in the Royal Navy, and commends itself for the greater simplicity it allows in working problems in spherical trigonometry. The other method in use is explained so that gentlemen, who are already provided with Norie's tables, can read this book without being obliged to purchase others.

I must conclude by remarking that the study of navigation requires considerable application; that a great deal of dry reading must be gone through; and that constant practice in working the problems presented is the only way to obtain a command of the subject.

Yachtsmen love the sea, but many of them are content to potter about the Channel or in close waters. I hope it may be my privilege, by showing them how to find paths in the trackless ocean, to inspire in them a love of deep sea cruising, and so induce them to experience for themselves the exhilarating delight of sailing, say, in the trade wind regions, where the deep blue sea sparkles and foams under the influence of the life giving wind and the beams of a cloudless sun.

CHAPTER II.

ALGEBRA.

ALGEBRA may be defined as a branch of mathematics in which letters represent numbers, and signs are used to denote the relations of such numbers to each other. One of the principal advantages to be derived from the use of algebraical letters and signs is that letters representing unknown quantities may be dealt with as if such quantities were known. It is customary to represent known quantities by the first letters of the alphabet, as a, b, c, &c., and unknown quantities by the last letters, as x, y, z.

The signs used in algebra, and in trigonometry, are $+, -, \times, \div, =, \sim, >, <, \therefore, \because$, and the signs of involution and evolution.

(1.) $+$ means *plus*, and is the sign of addition. Thus $a + b$ means that the number represented by b is to be added to that represented by a. Therefore, if a represents 10, and b represents 5, $a + b$ will equal 15. Again, $a + b + c$, &c., means that the numbers represented by a, b, and c, are to be added together, and so on.

(2.) $-$ stands for *minus*, and is the sign of subtraction. Thus, $a - b$ means that the number represented by b is to be subtracted from that represented by a. Therefore, if

a and b represent 15 and 10 respectively, $a - b$ will equal $15 - 10$; that is, 5. Again, if we have an expression such as $a - b + c - d$, it means that the numbers represented by a and c are to be added together, and the numbers represented by b and d are to be subtracted from the sum of a and c. Thus, if a represents 10, b 8, c 6, and d 4, the expression $a - b + c - d$ will be equal to $10 + 6$, that is 16, less $b + d$, or 12; in other words, $16 - 12$, the value, therefore, being 4. The rule then is, that the value of any such algebraical expression is the difference between the numbers represented by letters having *plus* signs, and those having *minus* signs. When brackets are used, as we shall see hereafter, this rule is modified.

(3.) × means *multiplied by*, and is the sign of multiplication. Thus, $a \times b$ means that the numbers represented by a and b respectively are to be multiplied together. Therefore, if a represents 6 and b 5, $a \times b$ will equal 6×5, that is 30, and 30 is called the *product* of a and b, or 6 and 5. Again, $a \times b \times c$, &c., means that the product of the numbers representing a and b is to be multiplied by the number representing c, and so on. $a \times b$ may be written with a dot, thus, $a . b$, but care must be taken not to confuse this dot with a decimal point. The latter is usually placed higher than the former. $a \times b$ is nearly always written ab, $a \times b \times c$, abc, and so on.

(4.) ÷ stands for *divided by*, and is the sign of division. Thus, $a \div b$ means that the number represented by a is to be divided by the number represented by b. Now, if a represents 1000 and b 5, $a \div b$ will equal 1000 divided by 5, or 200. $a \div b$ is usually written in a fractional form, thus $\frac{a}{b}$, and means the same thing.

(5.) $=$ means *equal to,* and signifies that any numbers between which it is placed are equal. Thus, if a represents 5 and b 10, then $2 \times a$ or $2a = b$, that is $10 = 10$, which is self-evident.

(6.) $\sim$ means the *difference* between any two numbers between which it is placed. It is convenient sometimes, when we have two unknown quantities, and do not know which is the greater, to use this sign for expressing the difference. Thus, if x represents 12 and y 20, the expression $x - y$ would be an impossible quantity, $12 - 20$; but $x \sim y$, or the difference between 12 and 20, would be 8.

It may be well, however, to mention in this connection that *minus* quantities are used in algebraical equations. For example, we will suppose that a merchant makes a loss during a certain year of £200, it would be algebraically correct to say that he had gained $-$ £200.

(7.) $>$ means greater than. Thus, $a > b$ means that a is greater than b.

(8.) $<$ means *less than.* For instance, $a < b$ signifies that a is less than b.

(9.) $\therefore$ denotes *then* or *therefore.*

(10.) $\because$ denotes *since or because.*

(11.) A small figure written above and to the right of another figure is the sign of involution. 8^2 means that 8 is multiplied by itself, or, in other words, raised to its second power. $8 \times 8 = 8^2$; $8 \times 8 \times 8 = 8^3$, or 8 raised to its third power; $8 \times 8 \times 8 \times 8 = 8^4$, or 8 raised to its fourth power, and so on.

(12.) $\surd$ is the sign of evolution. Thus, $\sqrt{8 \times 8} = 8$, or, in other words, the square root of 64 is 8; $\sqrt[3]{8 \times 8 \times 8} = 8$, or, the cube root of 512 is 8;

$\sqrt[4]{8 \times 8 \times 8 \times 8} = 8$, or the fourth root of 4096 is 8, and so on.

(13.) When several numbers are to be taken as forming one expression they are inclosed in brackets. Thus, supposing we want to multiply the expression $a+b$ by $x+y$, it may be written thus: $(a+b) \times (x+y)$ or $(a+b)(x+y)$. Sometimes, instead of using brackets, a line called a vinculum is drawn over the quantities to be bracketed. Thus, $(a+b)\ (x+y)$ may be written $\overline{a+b} \times \overline{x+y}$.

In such an expression as $(a+b+c) + (d-e)$ the brackets may be removed without altering its value, the signs remaining the same.

$$\therefore (a+b+c) + (d-e) = a+b+c+d-e.$$

But suppose we are dealing with the following expression: $(a+b+c)-(d-e)$, it will be necessary, if we remove the brackets, to change all the signs of the quantities inclosed in any brackets which are preceded by a minus sign, thus,

$$(a+b+c)-(d \overset{*}{-} e) = a+b+c-d \overset{*}{+} e,$$

and the reason for this is evident. The amount to be subtracted from $a+b+c$ is $d-e$ or d less e. But if we remove the bracket inclosing these quantities without changing the sign, we actually subtract from the first expression the amount d and e added together instead of the difference between them. Let us substitute figures for the letters by supposing that $a=10$, $b=8$, $c=5$, $d=7$, and $e=4$. Then we have:

$$\begin{array}{r}
(a+b+c)-(d-e) = (10+8+5)-(7-4) \\
(a+b+c) = 10+8+5 = 23 \\
(d-e) = 7-4 = \underline{\ \ 3} \\
\therefore (10+8+5)-(7 \overset{*}{-} 4) = 20
\end{array}$$

or, removing the brackets:

$$10+8+5-7 \overset{*}{+} 4 = 20$$

The rule then is, *that in removing brackets the signs of all quantities which are inclosed in a bracket preceded by a minus sign must be reversed.*

ADDITION.

Addition in algebra, when the terms to be added together are unlike, is performed by writing such terms one after the other, each with its proper sign before it. Thus, if we wish to add together the terms a, b, c, d, and e, we should write them down thus, $a + b + c + d + e$. Or, supposing it is required to add together $(a + b - c)$ and $(d - e)$, we should simply place the plus sign before the second expression, thus: $(a + b - c) + (d - e)$.

When any of the terms are alike, their sum is found by adding their co-efficients together, thus: $1a + 1a = 2a$, the co-efficients 1 and 1 being added together. When no co-efficient precedes an algebraical expression, the number 1 is understood. Again, $a + 2a + 3a$ would equal $6a$, and so on.

Example: add $9a + 10b$ to $3a + b$; the sum is $12a + 11b$, for the $9a$ and the $3a$ together make $12a$, and the $10b$ and the $1b$ together make $11b$.

When like terms have different signs, their sum is found by taking the difference between the sum of the co-efficients having plus signs and the sum of those having minus signs, and placing before it the sign of the greater sum, thus:

$$7a - 10a + 12a - a = 8a$$

because

$$7 + 12 = + 19 \text{ and } - 10 - 1 = - 11, \; 19 - 11 = 8a.$$

Again

$$(9a - 7b) + (8b - 10a) = b - a, \text{ for } + 8b - 7b = b, \text{ and } + 9a - 10a = - a.$$

SUBTRACTION.

Suppose we want to subtract a from $b + c$ it would be written $b + c - a$.

Again, if we wish to take $a + b$ from c, we shall have $c - (a + b)$, or taking away the brackets (13.), $c - a - b$.

In short, the best rule is to change the sign of every term to be subtracted, and proceed as in addition.

Example: Take $10a^2 + 12ab - 7b^2$ from $15a^2 - 7ab + 10b^2$.

$$\begin{array}{r} 15a^2 - 7ab + 10b^2 \\ -10a^2 - 12ab + 7b^2 \\ \hline \end{array}$$

and we have: $$5a^2 - 19ab + 17b^2$$

MULTIPLICATION.

We have already seen that the product of any letters denoting numbers may be written without any sign between them. Thus, abc means the product of a, b, and c, or $a \times b \times c$, and, of course, it is immaterial whether it is written abc, bca, or cab.

Suppose it is required to multiply $a + b$ by c, we must multiply each term of the first by c; $\therefore$ $(a + b) \times c$, or $(a + b)\,c = ac + bc$.

Again, $(a - b) \times c$, or $(a - b)\,c = ac - bc$.

To multiply $a + b$ by $c + d$ we proceed as follows:

$$\begin{aligned} (a + b)(c + d) &= (a + b)\,c + (a + b)\,d \\ &= ac + bc + ad + bd \end{aligned}$$

Again, to multiply $a + b$ by $c - d$.

$$\begin{aligned} (a + b)(c - d) &= c\,(a + b) - d\,(a + b) \\ &= (ca + cb - (da + db) \end{aligned}$$

or, taking away the brackets (13.)

$$= ca + cb - da - db.$$

(We must note here that if we multiply or divide a plus quantity by a plus quantity the product will also be plus.

If we multiply or divide a plus quantity by a minus quantity, or *vice versâ*, the product will be minus. And if we multiply or divide a minus quantity by a minus quantity the product will be plus.).

Now, to multiply $(a - b)\ (c - d)$ we have

$$\begin{aligned}(a - b\ (c - d) &= c\ (a - b) - d\ (a - b)\\ &= (ca - cb) - (da - db)\\ &= ca - cb - da + db.\end{aligned}$$

It will thus be seen that the multiplication of two bi-nomial expressions is effected by multiplying each term of the multiplicand by each term of the multiplier, and prefixing to these products the plus sign, if the factors have similar signs, and the minus sign if they are unlike. Let us again consider the last problem.

$$\begin{array}{l} a - b \\ \underline{c - d} \\ ac - cb - da + db \end{array}$$

$a \times c = ac$, both factors plus.

$-b \times c = -bc$, one factor plus, one minus.

$a \times -d = -ad$, one factor plus, one minus.

$-b \times -d = +bd$, both factors minus.

The rule thus indicated for multiplying bi-nomial expressions is applicable to all algebraical expressions having any number of terms.

Next let us consider the value of the expression $(a + b)^2$, that is $(a + b)\ (a + b)$

$$\begin{array}{l} a + b \\ \underline{a + b} \\ a^2 + ab \\ \underline{\quad + ab + b^2} \\ a^2 + 2ab + b^2 \end{array}$$

and similarly $(a - b)^2 = a^2 - 2ab + b^2$,

also $(a + b)\ (a - b) = a^2 - b^2$.

I would ask the reader to work these two latter problems, so that he may satisfy himself that he understands the explanation given.

Division.

As the product of the numbers denoted by a and b becomes ab, it is evident that if we divide ab by b or $\frac{ab}{b}$ the quotient will be a; also that $\frac{ab}{a} = b$, $\frac{abc}{a} = bc$, $\frac{abc}{ab} = c$, and so on. I do not think it necessary to go more fully into this branch of the subject, as the student may readily, by studying the above illustrations, and also inverting the examples worked out under the head of multiplication, gain sufficient knowledge.

Simple Equations.

We will now consider some of the properties of simple equations:

$$\text{Let } a + b = c.$$

This is called an equation because one side is equal to the other. That being so, if the same amount be subtracted from each of the equal parts, it is evident that the remainders will also be equal. Let us take b from each. Now we have

$$a + b - b = c - b$$
$$\text{or, } a = c - b \text{ since } + b \text{ and } - b \text{ go out.}$$

Now compare the two equations, and we find that $+ b$ on the left hand side has become $- b$ on the right hand side. Hence the rule that *any quantity on one side of an equation can be transferred to the other side by altering its sign.* It must be remembered, however, that a number of quantities bracketed when transferred, will only require the sign before the bracket to be changed, the signs of the quantities inclosed in the brackets remaining unaltered. Thus:

$$\begin{gathered}\text{Let } a + (b + c) = d \\ \text{then } a = d - (b + c), \\ \text{or, let } a - (b + c) = d \\ \text{then } a = d + (b + c).\end{gathered}$$

In the following equations we will consider such as involve multiplication and division.

$$\text{Let } a \times c = d.$$

Divide both sides by c, and we have

$$a = \frac{d}{c}.$$

When quantities are bracketed the same effect follows. Thus:

$$\begin{gathered}\text{Let } (a + b)\, c = (x + y)\, z \\ \text{then } c = \frac{(x + y)\, z}{a + b} \\ \text{or } \frac{(a + b)\, c}{x + y} = z \\ \text{or again, } (a + b = \frac{(x + y)\, z}{c} \\ \text{and again, } \frac{(a + b)\, c}{z} = (x + y)\end{gathered}$$

My readers would do well very carefully to note the above properties of equations, as we shall be continually using similar processes in considering trigonometrical formulæ. They are founded on the well-known axioms that equal things added to, subtracted from, or divided by other equal things remain equal.

CHAPTER III.

DECIMAL FRACTIONS.

IN ordinary arithmetic the value of any digit or figure in a number depends upon its position in such number. For instance, in the number 4378, the $8=8\times 1$, the $7=7\times 10$, the $3=3\times 10\times 10$, and the $4=4\times 10\times 10\times 10$. Now we will place a dot called a decimal point after the 8, and add some more figures thus: 4378·9216. The value of the portion of this expression before or to the left of the decimal point remains unchanged; that portion which follows it is called a decimal fraction, and the values of the figures composing it are as follows: the $9 = 9 \div 10$, the $2=2\div(10\times 10)$, the $1=1\div(10\times 10\times 10)$, and the $6 = 6 \div (10 \times 10 \times 10 \times 10)$. Or, in other words, $4378{\cdot}9216=4378\,\frac{9216}{10000}$. In a number containing decimal fractions, therefore, the figure immediately before the decimal point expresses units, and all the figures to its left are products of 10 and multiples of 10: while all the figures to the right of the decimal point are divided by 10, and by multiples of 10. Thus the expression $4378{\cdot}9216=\underline{4\times 1000}+\underline{3\times 100}+\underline{7\times 10}+\underline{8}+\underline{9\div 10}+\underline{2\div 100}+\underline{1\div 1000}+\underline{6\div 10000}$.

The great advantage to be derived from the use of decimal fractions in arithmetical operations is that the fractional part may be treated as a whole number, the placing of the decimal point determining the value of the result.

It is impossible to express many vulgar fractions as decimal fractions exactly, though by extending the decimal the difference between them may be made infinitely small.

To Convert Vulgar Fractions into Decimals.

Rule.—Add zeros to the numerator and divide by the denominator of the vulgar fraction to be converted into a decimal fraction, placing the decimal point, as shown in the following examples:

(1) To convert $\frac{1}{2}$ into a decimal fraction:

$$\begin{array}{r} 2)\underline{1\cdot0} \\ \cdot5 \end{array} \quad \therefore \tfrac{1}{2} = \cdot5 = \tfrac{5}{10} = \tfrac{1}{2}.$$

(2) To convert $\frac{1}{4}$ into a decimal fraction:

$$\begin{array}{r} 4)\underline{1\cdot000} \\ \cdot25 \end{array} \quad \therefore \tfrac{1}{4} = \cdot25 = \tfrac{25}{100} = \tfrac{1}{4}.$$

(3) To convert $\frac{1}{8}$ into a decimal fraction:

$$\begin{array}{r} 8)\underline{1\cdot0000} \\ \cdot125 \end{array} \quad \therefore \tfrac{1}{8} = \cdot125 = \tfrac{125}{1000} = \tfrac{1}{8}.$$

(4) To convert $\frac{2}{15}$ into a decimal fraction:

$$\begin{array}{r} 15)\underline{2\cdot0000} \\ \cdot13333, \&c. \end{array}$$

It is evident in this last example that the 3's in the decimal might be continued to infinity. Such an expression is called a recurring decimal, and is written with a dot over the recurring figure, thus: $\frac{2}{15} = \cdot1\dot{3}$.

(5) To convert $\frac{1}{75}$ into a decimal fraction :

$$\begin{array}{l} 75)1{\cdot}0000000({\cdot}01\dot{3} \\ \quad\;\; \underline{75} \\ \quad\;\;\; 250 \\ \quad\;\;\; \underline{225} \\ \quad\;\;\;\; 250 \end{array}$$

$$\text{and } \tfrac{1}{75} = {\cdot}01\dot{3}$$

In this case, when one zero is added to the numerator, the denominator will not go into it ; we, therefore, place a zero after the decimal point in the quotient and add another zero to the numerator, which is then divisible by the denominator.

For navigation and nautical astronomy it is often necessary to take the decimal values of seconds and minutes of time or arc in regard to the hours or degrees.

For example, we wish to convert 10 hours 12 minutes 30 seconds into the decimals of an hour. We first find what decimal of a minute is 30 seconds. There being 60 seconds in a minute we divide the number of seconds by 60, and have $\begin{array}{l} 60\underline{)30{\cdot}0} \\ \quad\;\; {\cdot}5 \end{array}$ $\therefore$ 30 seconds $=$ ·5 minutes. Now we have 12·5 minutes to convert into the decimal of an hour. We must, of course, divide this by 60, because there are 60 minutes in an hour, and we have $\begin{array}{l} 60\underline{)12{\cdot}5} \\ \quad\;\; {\cdot}208\dot{3} \end{array}$ and the result is that 10 hours 12 minutes 30 seconds $=$ 12·208$\dot{3}$ hours.

Here are some more examples :—

(6) Convert 27° 32′ 24″ into degrees and decimals of a degree.

$$\begin{array}{ll} 60\underline{)24''{\cdot}0} & \\ 60\underline{)32'{\cdot}4} & \text{or } 24'' = {\cdot}4'. \\ \quad\;\; {\cdot}54 & \text{or } 32'\ 24'' = {\cdot}54^\circ \end{array}$$

$$\therefore 27^\circ\ 32'\ 24'' = 27^\circ{\cdot}54.$$

(7) Convert 10 hours 40 minutes 28 seconds into hours and decimals of an hour.

$60)\underline{28''}$

$60)\underline{40{\cdot}46}$ or $28'' = 4\dot{6}'$

·6744 nearly, or 40min. 28sec. = ·6744 hours very nearly.

∴ 10hrs. 40min. 28sec. = 10·6744 hours very nearly.

In this example the decimal fraction is not strictly accurate, but near enough for all practical purposes. It must always be borne in mind that the fourth figure to the right of the decimal point is of very small fractional value, ·0001 of an hour being only equal to about one-third of a second, therefore it is never necessary to use more than four places of decimals except in logarithms, and in nearly all our work two places will be more than sufficient.

In reducing seconds to decimals of a minute, and minutes to decimals of an hour, or a degree, the easiest plan is to divide both divisor and dividend by 10, making the former 6, and in the latter moving the decimal point one place to the left. Thus in example 6 and 7 above:

Example (6)

$6{,}0)\underline{2{,}4''}$

$6{,}0)\underline{3{,}2'{\cdot}4}$

$\cdot 54^\circ$

Example (7)

$6{,}0)\underline{2{,}8''}$

$6{,}0)\underline{4{,}0'{\cdot}4\dot{6}}$

$\cdot 674\dot{3}^\circ$

this saves the labour of dividing by 60, and is quite simple.

ADDITION.

The addition of decimal fractions is performed by placing *the decimal points of the numbers to be added under one* *another* and proceeding as in ordinary arithmetic. Example: Add together 1·796, 10·5, ·0019 and 57.

$$\begin{array}{r} 1{\cdot}796 \\ 10{\cdot}5 \\ {\cdot}0019 \\ 57{\cdot} \\ \hline 69{\cdot}2979 \end{array}$$

and this is the sum required.

Subtraction.

Place the decimal points under one another and proceed as in ordinary arithmetic. Example: Subtract 9·167 from 112·4:

$$\begin{array}{r} 112{\cdot}4 \\ 9{\cdot}167 \\ \hline 103{\cdot}233 \end{array}$$

It is evident that the value of any decimal fraction is not affected by any number of zeros placed to the right of the decimal point, because as decimals represent the numerator of a fraction whose denominator is 10, or a multiple of 10, if the numerator is zero the fraction has no value, thus: ·001 = $\frac{1}{1000}$· ·0010 = $\frac{10}{10000} = \frac{1}{1000}$, and so on; and ·000 is of no value. Therefore in subtracting one decimal fraction from another, when there is no figure in the decimal in the top line, a zero is understood. For instance, subtract ·001 from 1.

$$\begin{array}{r} 1{\cdot}000 \\ {\cdot}001 \\ \hline {\cdot}999. \end{array}$$

Multiplication.

The rule is to proceed as in ordinary arithmetic, and point off in the product as many decimals as the sum of the decimals in the numbers multiplied together. Examples:

(1) Multiply 10·6 by 9·09
(2) „ 106·1 „ ·909
(3) „ ·0106 „ ·99.

(1.)	(2.)	(3.)
10·6 = 1 decimal	106·1 = 1 decimal	·0106 = 4 decimals
9·09 = 2 decimals	·909 = 3 decimals	·99 = 2 decimals
954 3	9549 4	954 6
954	9549	954
96·354 = 3 decimals.	96·4449 = 4 decimals.	·010494

In Example 1 there are three decimal places in the two numbers to be multiplied together, we, therefore, point off three decimal places in the product: in Example 2 there are four, we, therefore, point off four: and in Example 3 there are six, but as the product only gives us five figures we have to place a *zero to the left* of the product in order to get the right number of decimals after the decimal point.

DIVISION.

Division of decimals is performed in exactly the same way as in ordinary arithmetic. The decimal point is placed in the quotient so as to make the total number of decimal places in the divisor and quotient equal to the number of decimal places in the dividend. Examples:

(1) Divide 344·5194 by 1·9
(2) „ 6·885 „ ·009
(3) „ ·383773 „ 72·41
(4) „ 1 „ ·0125

(1)

```
1·9)344·5194(181326
    19
    154
    152
      25
      19
        61
        57
          49
          38
           114
           114
```

In this example there are four decimal places in the dividend and one in the divisor; there must, therefore, be three in the quotient, and the decimal point is placed to the left of the third figure from the right, in this manner: 181·326.

(2)

```
·009)6·885
      765
```

Here there are three decimal places in the divisor and three in the dividend; there are none, therefore, in the quotient, which is 765.

(3)

```
72·41)·383773(53
       36205
        21723
        21723
```

Here we have six decimal places in the dividend and only two in the divisor; there must then be four in the quotient, and we obtain these by placing two zeros *before the quotient* obtained, thus : ·0053.

$$\begin{array}{c} (4) \\ \cdot0125)1\cdot0000(80 \\ \underline{1\cdot000} \\ 0 \end{array}$$

In cases such as this it is best to add a sufficient number of zeros to the dividend to give it the same number of decimals as the divisor has ; then divide as in ordinary arithmetic, and the quotient will be a whole number, which in this example is 80.

In the problems to be considered later on, the use of decimal fractions is in many cases indispensable.

CHAPTER IV.

LOGARITHMS.

THE logarithm of a number is the power to which a certain number called the base must be raised to produce that number. The base generally employed is the number 10. Suppose we want to find the logarithm of 100, the base being 10. Thus 10×10 or 10^2, in other words, 10 to the power of 2, is equal to 100; therefore, the logarithm of 100 to the base 10 is 2. It is usually written in this way, the logarithm of $100_{10} = 2$. Similarly the logarithm of $1000_{10} = 3$, because $1000 = 10^3$, or 10 to the power of 3. As all the books of logarithms which we shall use are constructed on the base of 10, we may restrict ourselves to the consideration of such logarithms only, and accordingly the small 10 denoting the base will be omitted.

$$
\begin{array}{ll}
\text{Now } \overline{10 \times 10} \times \overline{10 \times 10 \times 10} & = 10^5 \\
\text{but } 10^2 \times 10^3 & = 10^5 = 10^{2+3} \\
\text{and 2 is the logarithm of } 10^2 & \\
\text{and 3 is the logarithm of } 10^3 & \\
\therefore 2 + 3 = \text{the logarithm of } (10^2 \times 10^3)\text{, or } 10^5. &
\end{array}
$$

This demonstrates that in multiplication the *logarithm of the product is equal to the sum of the logarithms of its factors*; or, in other words, to multiply two numbers together we have only to add together the logarithms of such numbers.

Again, if we want to divide 1000 by 100, we know that the logarithm of 1000 is 3, and that of 100 is 2; but $1000 \div 100 = 10$ to the first power, and the

$$\text{logarithm of } \frac{1000}{100} = 10^{3-2} = 10^1,$$

so that in division *the logarithm of the quotient is equal to the logarithm of the dividend less the logarithm of the divisor*; that is to say, that division by the use of logarithms is performed by subtracting from the logarithm of the number to be divided the logarithm of the number dividing.

The logarithm of 1 to any base is 0, and we can prove this as follows:

$$\frac{100}{100} = 1, \text{ the logarithm of } \frac{100}{100} = 2 - 2 = 0$$

$$\therefore \text{ the logarithm of } 1 = 0$$

To raise a number to any given power, we multiply the logarithm of the number by the power to which it is to be raised.

The reason why we do this may be exemplified thus: suppose it is required to raise 10^2 to its fifth power, that is $10^2 \times 10^2 \times 10^2 \times 10^2 \times 10^2$, the product of these five quantities is 10^{10}, but this is also equal to $10^{2 \times 5}$. This rule applies to all similar problems.

Also, the extraction of roots or evolution is performed by dividing the logarithm of the number by the root required.

$$\text{Thus } \sqrt[5]{10^2} = 10^{\frac{2}{5}},\ \sqrt[3]{10^6} = 10^{\frac{6}{3}} = 10^2, \text{ and so on.}$$

It is obvious that the logarithms of all numbers not 10 or multiples of 10 must be fractional or mixed numbers.

Let the logarithm of any such number be 3·179245. The whole number 3 is called the *characteristic*, and the decimal fraction ·179245 is the *mantissa* of the logarithm. Also, that the logarithms of all numbers between 1 and 10 must be smaller quantities than 1; that the logarithms of numbers between 10 and 100 must be greater than 1 and less than 2; that the logarithms of numbers between 100 and 1000 must be greater than 2 and less than 3, and so on. Hence the rule that the characteristic of any logarithm is 1 less than the number of digits in the whole number of which it is the logarithm.

The logarithms of fractions have always minus characteristics; for if we divide, say, 10 by 100 by logarithms, we have $1 - 2 = -1$ as the logarithm of the quotient $\frac{1}{10}$. Hence the characteristic of the logarithms of fractions greater than $\frac{1}{10}$ is $\bar{1}$. (It is convenient to write the minus sign over the characteristics of logarithms.) The characteristic of logarithms of fractions less than $\frac{1}{10}$ and greater than $\frac{1}{100}$ is $\bar{2}$, and so on.

The logarithms of all numbers and trigonometrical ratios are calculated and tabulated in the various books of logarithms published. Let us now consider their practical use.

Suppose we wish to multiply 7267 by 4596, we find in the tables that the logarithm of 7267 (p. 414 in Inman; Norie, p. 101), is ·861355, but, as explained above, this is only the *mantissa* of the logarithm; we must insert its *characteristic*, which in this case, the number consisting of 4 digits, is 3. Therefore, the logarithm of 7267 is 3·861355, or, in other words, 10 to the power of 3·861355, or, $10^{3\cdot861355}$ is equal to 7267. Similarly the logarithm of 4596 is 3·662380 (Inman, p. 407; Norie, p. 96). Now in order to obtain the logarithm of the product of 7267 and 4596 we only have to add the logarithms of these two numbers together. Thus 3·861355 + 3·662380 = 7·523735. Now we find (Inman, p. 403 ; Norie, p. 94) that the next logarithm less than this is ·523616, and we proceed to take out the number wanted thus:

Logarithm of product required	7·523735
„ of 3339	·523616
	119
Difference for 9 =	117
	20
„ „ 1 =	13
	70
„ „ 5 =	65
	50
„ „ 3 =	39
	110
„ „ 8 =	104
	6

So that we find the product to be 333991538, but the characteristic of the logarithm of this number is 7; we want, therefore, 8 digits in the number, because we are now finding the number corresponding to the logarithm, the reverse of what we did before, and we count these from the left of the number found, and place a decimal point. The number, therefore, will be 33,399,153·8.

This result, it will be observed, is fractionally incorrect, and that is caused by the logarithms only being carried to six places of decimals. But in practice we shall never have to deal with such enormous figures as the above, and for smaller amounts the tables as constructed give almost perfectly accurate results. Multiplied arithmetically 7267 and 4596 give a product of 33,399,132, which shows the result obtained by logarithms to be 22 in error in over 33,000,000, which is practically nothing.

Division is performed by subtracting the logarithm of the divisor from the logarithm of the dividend, and thus obtaining the logarithm of the quotient. For example, it is required to divide 7289 by 343.

Logarithm of 7289 = 3·862668 (Inman, 414; Norie, 101)
" 343 = $\underline{2{\cdot}535294}$ (" 403; " 95)
" of quotient 1·327374 (" 400; " 92)

$$\therefore \frac{7289}{343} = 21{\cdot}25 \text{ very nearly.}$$

Again, let us divide 7 by 160.

$$\text{Logarithm of } \frac{7}{160} = \text{logarithm } 7 - \text{logarithm } 160,$$

and is evidently a fraction.

Logarithm of 7 = 0·845098 (Inman, 395; Norie, 90)
160 = $\underline{2{\cdot}204120}$ (" 395; " 92)

$\therefore$ " of $\frac{7}{160} = \bar{2}{\cdot}640978$

Let us take this out of the tables (Inman, 406; Norie, 96).

Logarithm of $\frac{7}{160} = \bar{2}{\cdot}640978$ the mantissa of which is the logarithm of 4375.

Now, the characteristic, $\bar{2}$, shows that the fraction, the logarithm of which it is a part, must be less than $\frac{1}{10}$, and more than $\frac{1}{100}$. Therefore, we have to make the number found a decimal fraction whose value is between these amounts. Thus, ·04375 is the number required. This fraction, reading $\frac{4375}{100000}$, is less than $\frac{10000}{100000}$, and more than $\frac{1000}{1000000}$, which fractions are respectively equal to $\frac{1}{10}$ and $\frac{1}{100}$.

In working out the problems which we shall consider in future papers, my readers will gain experience in the use of logarithms. To obtain facility, however, in the use of the tables, it would be well for them to work out examples of multiplication, division, involution, and evolution, which they can set for themselves, and the results of which they can prove by common arithmetic.

EXAMPLES.

(1) Multiply 101 by 796:

Logarithm 101 = 2·004321 (Inman, p. 397; Norie, p. 91).
Logarithm 796 = 2·900913 („ p. 416; „ p. 102).
Logarithm (101 × 796) = 4·905234
Logarithm 8039 = 905202 (Inman, p. 417; Norie, p. 102).
Difference for 6 = 32 (Inman, p. 417).

The product, therefore, is 80396.

(2) Multiply 8·02 by 10·59.

Logarithm 8·02 = 0.904174 (Inman, p. 417; Norie, p. 102).
Logarithm 10·59 = 1·024896 („ p. 397; „ p. 91).
Logarithm (8·02 × 10·59) = 1·929070
Logarithm 8493 = 929061 (Inman, p. 418; Norie, p. 103).
·9
Difference for 1 = 5 („ „)
40
Difference for 8 = 41 („ „)

The product, therefore, is 84·9318. The characteristic being 1, the decimal point is placed two figures from the left.

(3) Divide 1242 by 9:

Logarithm 1242 = 3·094122 (Inman, p. 397; Norie, p. 91).
Logarithm 9 = 0·954242 (,, p. 419; ,, p. 104).
Logarithm (1242 ÷ 9) = 2·139880 (,, p. 398; ,, p. 91).

The quotient, therefore, is 138.

(4) Divide 996·53 by 45·4:

Logarithm 996·5 = 2·998477 (Inman, p. 422; Norie, p. 105).
parts for 3 = 13
Logarithm 996·53 = 2·998490
Logarithm 45·4 = 1·657056 (Inman, p. 407; Norie, p. 96).
Logarithm (996·53 ÷ 45·4 = 1·341434 (,, p. 400; ,, p. 92).

The quotient, therefore, is 21·95.

Full explanation of the practical use of the Tables is given in both Inman and Norie, so I shall not deal further with the subject of logarithms here.

CHAPTER V.

PLANE TRIGONOMETRY.

THE word trigonometry is derived from two Greek words, signifying "I measure a triangle."

Plane trigonometry has reference to problems connected with plane triangles—that is, to triangles described on a flat surface; and spherical trigonometry to the solution of triangles described on the surface of a sphere.

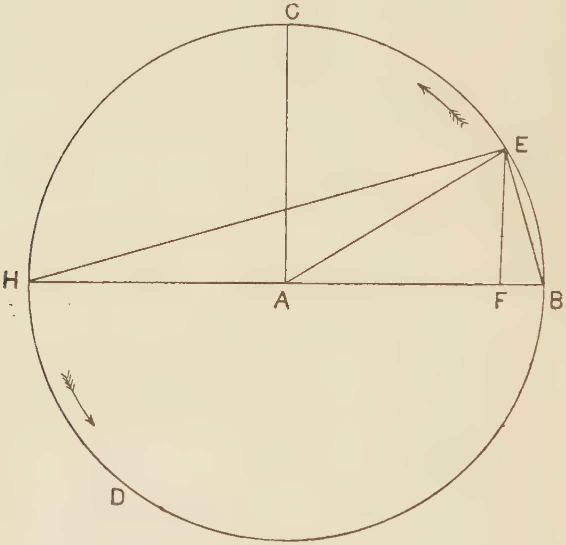

FIG. 1.

With the centre A, and radius A B, describe the circle B C D. The point B will have described an arc, B C D B, the circumference of a circle. This is divided into 360 equal portions, which are called degrees; there are 60 minutes in each degree, and 60 seconds in each minute of arc. Now suppose the radius had been stopped when it had completed one quarter of the circle, at C, it is evident that the arc described in that case would be a quarter of 360 degrees—that is, 90 degrees. The angle C A B is called a right angle, and the arc C B a quadrant. There are 90° in the angle C A B, and 90° in the arc C B. The length of a degree of arc varies directly with the length of the radius; or, in other words, the size of the circle depends on the length of its radius. On this fact is founded the formula that the circular measure of any angle is equal to $\frac{\text{arc}}{\text{radius}}$, and this is true with regard to all circles and all angles.

Let us suppose that the radius A B in its revolution was stopped at E. Draw E F perpendicular to A B. Here we have a triangle, A E F, of which the angle F is a right angle. This triangle is therefore called a right angled triangle. Join E B and E H.

An obtuse angle is an angle greater than a right angle. An acute angle is an angle less than a right angle. Thus, the angle C A H being a right angle, then the angle H A E is evidently an obtuse angle, and the angle C A E an acute angle. The triangles H E F, A E F, and B E F are right angled triangles, because the angles E F H and E F B are right angles.

Let A B C (Fig. 2) be a right angled triangle, of which the angle B is a right angle; then, with regard to the angle A, the side C B is called the perpendicular, the side A B the base. The side C A, opposite to the right angle B, is called the hypothenuse of the triangle.

With regard to angle C, A B is the perpendicular and C B the base. In all right angled triangles the side opposite the right angle is the hypothenuse, and the side opposite to either of the two acute angles is the perpendicular as regards that angle, and the side next to it the base; so that what is the base to one of these angles is the perpendicular to the other, and *vice versâ*.

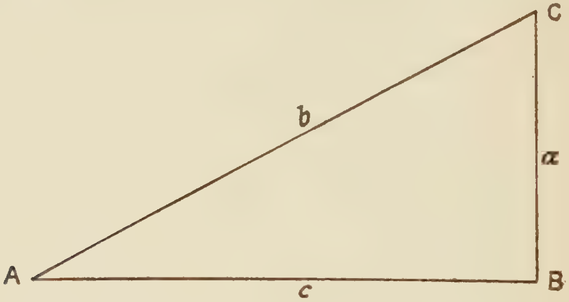

FIG. 2.

To facilitate the solution of trigonometrical problems, certain ratios have been determined. The trigonometrical ratios of any angle are the sine, cosine, tangent, cotangent, secant, cosecant, versine, coversine, and haversine.

$\frac{C\,B}{A\,C}$, that is $\frac{\text{perpendicular}}{\text{hypothenuse}}$, is called the sine of A.

$\frac{A\,B}{A\,C}$, that is $\frac{\text{base}}{\text{hypothenuse}}$, is called the cosine of A.

$\frac{C\,B}{A\,B}$, that is $\frac{\text{perpendicular}}{\text{base}}$, is called the tangent of A.

$\frac{A\,B}{C\,B}$, that is $\frac{\text{base}}{\text{perpendicular}}$, is called the cotangent of A.

$\frac{\text{A C}}{\text{A B}}$, that is $\frac{\text{hypothenuse}}{\text{base}}$, is called the secant of A.

$\frac{\text{A C}}{\text{C B}}$, that is $\frac{\text{hypothenuse}}{\text{perpendicular}}$, is called the cosecant of A.

When the cosine of A is subtracted from unity, the remainder is called the versed sine or versine of A. When the sine of A is subtracted from unity, the remainder is called the coversed sine or coversine of A. And half of the versine is called the haversine. So that

Versine = (1 — cosine).
Coversine = (1 — sine).
Haversine = $\frac{1}{2}$ (1 — cosine).

Sine, cosine, tangent, cotangent, secant, cosecant, versine, coversine, and haversine are usually abbreviated thus: Sin., cos., tan., cot., sec., cosec., vers., covers., and hav.

Degrees, minutes, and seconds of arc are generally written °, ′, ″; thus, 70 degrees 20 minutes 30 seconds would be 70° 20′ 30″.

The number of degrees that an acute angle is less than a right angle, or 90°, is called the complement of such angle. Thus the complement of A is 90°—A. The number of degrees that any angle is less than two right angles, or 180°, is called the supplement of such angle. Thus the supplement of A is 180°—A.

The three angles in any plane triangle are together equal to two right angles (Euclid, Book 1, Prop. 32). In the triangle A B C, B is a right angle, therefore the angles A and C must together equal another right angle; or A and C are the complements one of the other.

(For convenience we will call the sides opposite the angles A, B, C, *a*, *b*, *c*, respectively; so that the angles shall always be represented by capital letters, and the sides opposite to them by small letters, side a being opposite to angle A; side b to angle B; and side c to angle C.)

Now sin. $C=\frac{c}{b}$, but cos. $A=\frac{c}{b}$, therefore sin. C=cos. A. But A is the complement of C, therefore sin. C=cos. (90°−C). In a similar manner it may be shown that sin. A=cos. (90°−A), cos. A=sin. (90°−A), tan. A=cot. (90°−A), cot. A=tan. (90°−A), sec. A=cosec. (90°−A), and cosec. A=sec. (90°−A).

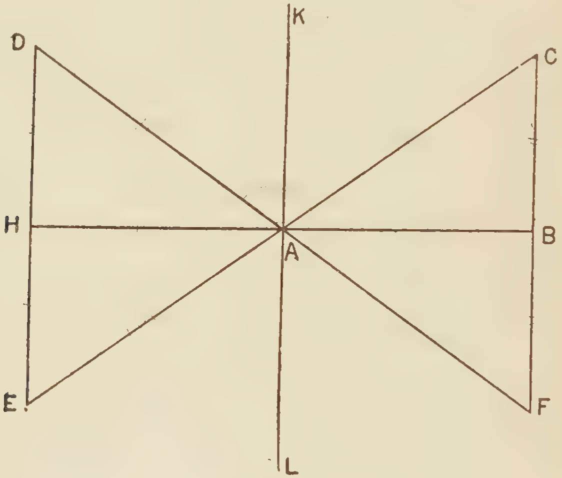

Fig. 3.

Let us consider for a few moments the value of trigonometrical ratios of angles greater than 90° with respect to those of angles less than 90°. Suppose that A B has revolved round the centre A in the direction C, D, E, &c. Let the triangles C A B, B A F, D A H, and H A E be similar in every respect, by making the sides C B, B F, D H, and H E equal, and the sides A B and A H equal, as also the sides A C, A F, A D, and A E. The angles are also equal to the corresponding angles in the other triangles, since their respective sides are equal (Euclid, Book 1, Prop. 8).

The angle D A B (Fig. 3) is an obtuse angle, and the angle D A H is its supplement. The sine of angle D A H is $\frac{DH}{AD}$, and this is also the sine of angle D A B because the trigonometrical ratios of an obtuse angle are determined by extending one of the sides forming part of such angle and dropping on the side thus extended a perpendicular line from the other side; so that with regard to the angle D A B, D H is the perpendicular, H A is the base, and A D is the hypothenuse. Similarly the trigonometrical ratios of angles greater than 180° and less than 270° are determined by the triangle H A E; and those of angles greater than 270° and less than 360° by the triangle A B F.

It must be noted that all the lines drawn above and perpendicular to H B are +; and that all lines drawn perpendicular to and below that line are −; that lines drawn to the right of and perpendicular to K L are +; that lines drawn to the left of and perpendicular to that line are −, and that the revolving line is always +. Therefore the lines D H, C B, A B, A C, A D, A E, and A F are +; and the lines H E, B F, and H A are −. Now the angle C A B is less than 90°. The angle B A D is more than 90° and less than 180°. Still going round the same way, the angle inclosed by the lines B A and A E is greater than 180° and less than 270°, and that between the lines B A and A F is greater than 270° and less than 360°. The sin. of C A B $= \frac{CB+}{AC+}$. The sin. of B A D =

$\frac{\text{D H}+}{\text{A D}+}$, but $\frac{\text{D H}}{\text{A D}} = \frac{\text{C B}}{\text{A C}}$, therefore sin. C A B = sin. D A H, or the sine of any angle is equal to the sine of its supplement, for D A H is the supplement of D A B and *vice versâ*. The cos. C A B = $\frac{\text{A B}+}{\text{A C}+}$ and the cos. D A H = $\frac{\text{A H}-}{\text{A D}+}$, giving a minus result, and showing that the cosine of any angle greater than 90° and less than 180° is a minus quantity, and that the cosine of any angle less than 90° is equal to the − cosine of its supplement. This fact will enable us, in working out the following problem, to know that the angle we shall find will be more than 90°. As the tables only give the logarithms of sines, cosines, &c., from 0° to 90°, therefore it becomes necessary to subtract the angle taken out of the tables from 180° to obtain the correct result. Thus $\cos.\ x = \cos.\ \overset{-}{\text{A}} \times \sin.\ \overset{+}{\text{B}}$. If A is greater than 90°, cos. A will be a minus quantity, sin. B being a plus quantity the product must therefore be minus, and the angle to be found will be between 90° and 180°.

The positive or negative values of any angles may be easily determined from this figure.

It is merely for convenience that the lines are called positive or negative according to their position with regard to the point A ; any other symbols would do as well ; but, as we use the + and − signs commonly, and as no confusion is caused by thus employing them, this method has been generally adopted.

We will now proceed to consider what practical use can be made of the trigonometrical ratios.

One side and an angle, or two sides, of a right angled plane triangle being known (excluding the right angle) it is easy to calculate the other parts. Suppose we are given (Fig. 2) a and A to find c. Cot. $A = \frac{c}{a} = \frac{\text{base}}{\text{perpendicular}}$; therefore, multiplying each side of this equation by a, we have

$$c = a \times \text{cot. A.}$$

Again, let a and b be known to find A.

$$\text{Sin. A} = \frac{a}{b};$$

then, having a and b, let us find c.

By Euclid (Book 1, Prop. 47)

$$b^2 = a^2 + c^2$$
$$\therefore c^2 = b^2 - a^2$$
$$\text{or, } c = \sqrt{b^2 - a^2}.$$

But we can solve this last problem by using the trigonometrical ratios thus: by finding first an angle and then the side required.

$$\text{(1) Sin. A} = \frac{a}{b}$$

$$\text{(2) Cos. A} = \frac{c}{b} \therefore c = b \times \text{cos. A.}$$

We find by (1) the angle A, and then by (2) the side required.

On the solution of right angled plane triangles depends all plane sailing, parallel sailing, traverse sailing, mid-latitude sailing, &c., so that the above examples should be carefully studied.

CHAPTER VI.

PLANE TRIGONOMETRY (*continued*).

THE only other formula in plane trigonometry which it will be necessary to investigate is, that the sides of any plane triangle are in direct proportion to the sines of the angles opposite to them.

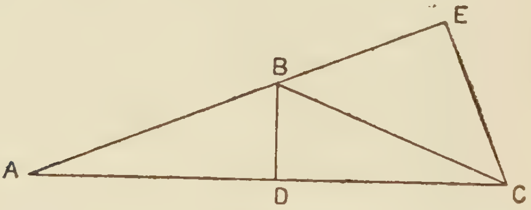

FIG. 4.

Let A B C (Fig. 4) be a plane triangle. Draw B D perpendicular to A C. Here we have two right angled triangles, A B D and C B D.

$$\text{Now, sin. } B\,A\,D = \frac{B\,D}{A\,B}, \text{ and sin. } B\,C\,D = \frac{B\,D}{B\,C},$$

$$\therefore \text{ sin. } B\,A\,D : \text{sin. } B\,C\,D :: \frac{B\,D}{A\,B} \cdot \frac{B\,D}{B\,C},$$

$$\text{and } \frac{\text{sin. } B\,A\,D}{\text{sin. } B\,C\,D} = \frac{\dfrac{B\,D}{A\,B}}{\dfrac{B\,D}{B\,C}} = \frac{B\,D\,.\,B\,C}{B\,D\,.\,A\,B} = \frac{B\,C}{A\,B},$$

or, the sines of these angles are directly proportional to their opposite sides.

Now, extend the side A B to join E C, making the angle A E C a right angle. We have seen that the sine of any angle is equal to the sine of its supplement. Therefore, sine A = sine (180°−A). In the right angled triangle B E C, sin. E B C $= \frac{E\,C}{C\,B}$, but angle E B C = 180°−angle A B C; it is therefore the supplement of angle A B C, and therefore sin. A B C $= \frac{E\,C}{C\,B}$, and sin. B A C $= \frac{E\,C}{A\,C}$; ∴ sin. A B C : sin. B A C :: A C : C B. By similarly extending C B, we can prove that sin. A B C : sin. B C A :: A C : A B. In any plane triangle, therefore, the sides are directly proportional to the sines of the angles opposite to them. Or, sin. A : sin. B : sin. C :: $a : b : c$.

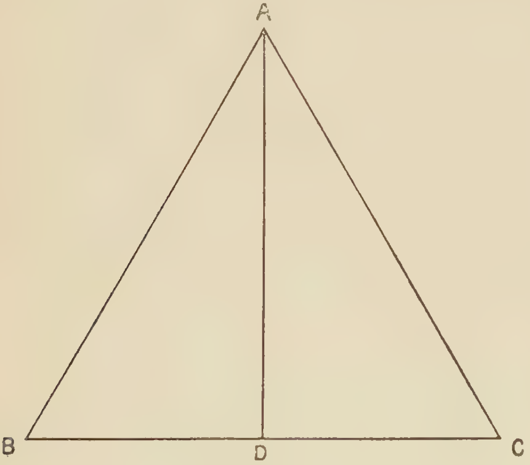

FIG. 5.

Let us consider briefly the numerical value of certain angles.

Let A B C (Fig. 5) be an equilateral triangle. Each of its angles is, therefore, equal to one-third of two right angles (180°), or 60°. From the point A, draw A D perpendicular to B C. We have now two right angled triangles, whose angles are respectively 90°, 60°, and 30°, because in the triangle A D B, the angle $A\,D\,B = 90°$, and $A\,B\,D = 60°$, $\therefore$ the third angle of the triangle, D A B, must equal 30°, the three angles of any plane triangle being together equal to 180°, and the triangle $A\,D\,C =$ the triangle A D B. The angle $D\,A\,C = D\,A\,B = 30°$; and $D\,C = B\,D = \frac{1}{2}\,B\,C$. Now $\sin.\ D\,A\,C = \frac{D\,C}{A\,C} = \frac{\frac{1}{2}\,B\,C}{A\,C} = \frac{1}{2}$, since $B\,C = A\,C$. But $D\,A\,C = 30°$, $\therefore \sin.\ 30° = \frac{1}{2}$. Again (Euclid, Book 1, Prop. 47), in the triangle A D C, $A\,C^2 = A\,D^2 + D\,C^2$; $\therefore A\,D^2 = A\,C^2 - D\,C^2$, or $A\,D = \sqrt{A\,C^2 - D\,C^2}$. Let $A\,D = 2$, and $D\,C = 1$, which is relatively correct. Then $A\,D = \sqrt{A\,C^2 - D\,C^2} = \sqrt{4 - 1} = \sqrt{3}$.

$$\text{Now sin. } A\,C\,D = \frac{A\,D}{A\,C}, \text{ or sin. } 60° = \frac{\sqrt{3}}{2} = \tfrac{1}{2}\sqrt{3}.$$

By considering these values and others easily deducible from them, we find that the values of half of the trigonometrical ratios are fractional. Thus the sines of all angles between 0° and 90°, not inclusive, are fractions. So are the cosines. The secants and cosecants, however, are greater than unity. The tangents of angles between 0° and 45° are fractions, and those from 45° to 90° are greater than unity ; and the reverse of this holds good with cotangents. The tables of log sines, &c. (Inman 32 ; Norie XXV.), also show these values. Now we have seen that the logarithms of quantities less than unity have a minus characteristic. To avoid the complications to which this would give rise in the practical use of logarithms, in the tables usually employed, ten is added to all the logarithms of trigonometrical ratios. This must always be borne in mind; and care should be taken to deduct the tens thus added when we have to look out for a logarithm of a number, and have been using the logarithms of sines, cosines, &c., in the calculation worked. Thus, if it is required to work the following expression, $a = b$ cos. A, we should proceed thus:

$$\text{Log. } a = \text{log. } b. + \text{log. cos. A} - 10,$$

so that we must subtract ten from the logarithm found by adding log. b and log. cos. A together.

I shall now endeavour to show the practical use of logarithms in the solution of problems in Plane Trigonometry.

In the right angled triangle, Fig. 2:

$$\text{Let } a = 107,\ b = 220 \text{ to find A}$$

$$\text{Sin. A} = \frac{a}{b}$$

$$\text{Log. sin. A} - 10 = \text{log. } a - \text{log. } b.$$

$$\text{or, Log. sin. A} = \text{log. } a - \text{log. } b + 10 = \text{log. } 107 - \text{log. } 220 + 10$$

log. 107 = 2·029384 (Inman, p. 395, Norie, p. 91)
log. 220 = 2·342423 („ p. „ „ p. 93)

$$\begin{array}{rr} & \bar{1}\cdot686961 \\ \text{add} & \underline{10\cdot000000} \\ \text{Log. sin. A} = & 9\cdot686961 \end{array}$$

∴ A = 29° 6′ 0″ nearly (Inman, p. 240, Norie, p. 188).

In examples such as the above it is better to add the 10 to the upper row of logarithms, thus:

$$\begin{array}{r} 12\cdot029384 \\ \underline{2\cdot342423} \\ 9\cdot686961 \end{array}$$

Let us now proceed to find the value of c.

$$\frac{c}{b} = \text{cos. A}, \therefore c = b \times \text{cos. A},$$

or, $\log. c = \log. b + \log. \text{cos. A} - 10 = \log. 220 + \log. \text{cos. } 29° \ 6' - 10$

$$\begin{aligned} \log. 220 &= 2{\cdot}342423 \\ \log. \text{cos. } 29° \ 6' &= \underline{9{\cdot}941398} \text{ (Inman, p. 240, Norie, p. 188)} \\ & 12{\cdot}283821 \\ \text{subtract } & \underline{10{\cdot}000000} \\ \text{Log. } c &= 2{\cdot}283821 \text{ (Inman, p. 399, Norie, p. 92),} \\ \text{and } c &= 192{\cdot}2 \text{ nearly.} \end{aligned}$$

In practice the 10 may be subtracted by striking out the digit 1 from the sum of the logarithms of b and cos. A.

In triangle A B C, Fig. 4, let B C = 1·78, A B = 2·175, and angle B A C = 19° 25′ 45″, to find angle B C A.

$$\frac{\text{Sin. B C A}}{\text{Sin. B A C}} = \frac{\text{A B}}{\text{B C}}$$

$$\therefore \text{sin. B C A} = \frac{\text{sin. B A C} \times \text{A B}}{\text{B C}}$$

$\therefore \text{Log sin. B C A} - 10 = \log. \text{sin. B A C} - 10 + \log. \text{A B} - \log. \text{B C}.$

-10 on each side goes out, and we have

$$\log. \text{sin. B C A} = \log. \text{sin. } 19° \ 25' \ 45'' + \log. 2{\cdot}175 - \log. 1{\cdot}78.$$

$$\begin{aligned} \text{Log. sin. } 19° \ 25' \ 45'' &= 9{\cdot}521976 \text{ (Inman, p. 201, Norie, p. 168)} \\ \text{Log. } 2{\cdot}175 &= \underline{0{\cdot}337459} \text{ (,, p. 400, ,, p. 92)} \\ & 9{\cdot}859435 \\ \text{Log. } 1{\cdot}78 &= \underline{0{\cdot}250420} \text{ (,, p. 395, ,, p. 92)} \\ \text{Log. sin. B C A} &= 9{\cdot}609015 \end{aligned}$$

Angle B C A = 23° 59′ 0″ (Inman, p. 219, Norie, p. 177.)

It will be well for my readers to work out the following problems by logarithms :

In the right angled triangle, Fig. 2:

Let $b = 210$, $A = 30°$, find c;
,, $c = 5001$, $a = 1249$, find C;
,, $a = 10{\cdot}81$, $C = 69° 10' 30''$, find b.

In the plane triangle A B C, Fig. 4:

Let $BC = {\cdot}789$, $AC = {\cdot}55$, $BAC = 62° 10' 0''$, find A B C.

CHAPTER VII.

DEFINITIONS AND ASTRONOMICAL DIAGRAMS.

THE earth being a sphere, all lines drawn upon its surface are portions of circles or curves. In practice it is found, however, that comparatively small areas on the globe may be treated as flat surfaces, and triangles drawn thereon may be solved by plane trigonometry.

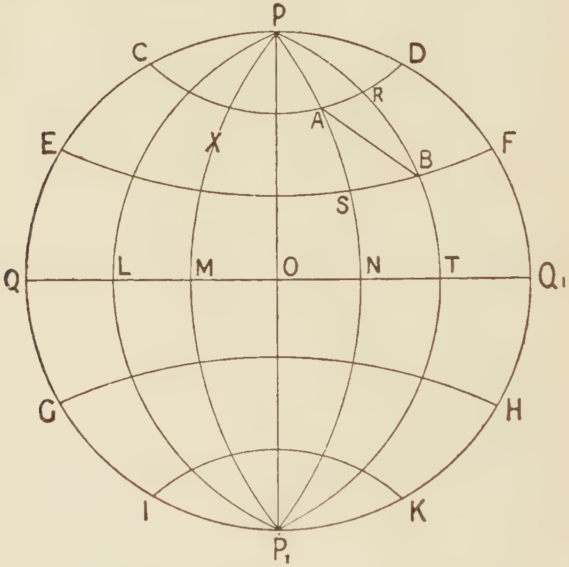

FIG. 6.

Let Fig. 6 represent the terrestrial globe. P and P_1 are the poles, Q Q_1 half of the equator; a straight line joining P and P_1, through the centre of the earth, is called the axis of the earth, round which she revolves; it is also the polar diameter. A straight line joining Q and Q_1 passing through the centre of the earth is an equatorial diameter; it is somewhat longer than the polar diameter, because the earth is slightly flattened at the poles. A circle drawn on a sphere whose plane passes through its centre is called a great circle. All other circles are called small circles.

A *terrestrial meridian* is a great circle passing through the poles, and cutting the equator at right angles. P N P_1, P M P_1, P L P_1, &c., are such meridians.

The *equator* is a great circle equidistant from the poles. Q O Q_1 is a part of the equator. Every part of the equator is 90° from either pole.

Parallels of latitude are small circles drawn parallel to the equator. E F, C D, G H, and I K are parallels of latitude.

The *meridian of a place on the earth* is that meridian which passes through the place.

The *first* or *prime meridian* is that fixed meridian from which the longitude of places on the earth is measured. The meridian of Greenwich is in most countries taken as the prime meridian.

The *longitude of a place* is the arc of the equator intercepted between the first meridian and the meridian of the place. It is named east or west according to its position with regard to the meridian of Greenwich, and cannot exceed 180°.

The *latitude of a place* is the arc of a meridian intercepted between the equator and the place, and is named N. or S., as it is north or south of the equator.

The *true course* of a vessel is the angle which her track makes with the terrestrial or true meridians.

The *correct magnetic course* is the angle between the direction of the ship's head and the magnetic meridian, or the line passing through the place of the ship and the magnetic pole.

The *compass course* is the direction of the ship's head by the standard compass.

The *difference of latitude* between two places is the arc of the meridian intercepted between their respective parallels.

The *difference of longitude* between two places is the arc of the equator intercepted between their respective meridians.

A *nautical mile* is equal to the mean length of a minute of latitude, and is very nearly 6080ft.

A *rhumb line* on a sphere is a curve which cuts all meridians at the same angle. The true course of a ship is a rhumb line.

The *distance* between two places is the length of the rhumb line joining them expressed in nautical miles.

The *departure* is the actual easting or westing made in nautical miles.

The *middle latitude* of two places on the same side of the equator is half the sum of their latitudes. If they are on opposite sides it is half their difference marked N. or S., as the greater latitude is N. or S.

The ship is said to describe a *traverse* when she sails on two or more courses.

A *Mercator's chart* is constructed on the following principles:

All the meridians are drawn parallel to each other, so that a rhumb line on a Mercator's chart is a straight line, because it cuts each meridian at the same angle. The distance between parallels of latitude is increased in the same proportion as the distance between the meridians on the chart becomes greater than is due to their position on the surface of the earth. Thus in latitude 60° on the globe the distance between any two meridians 1° apart is thirty nautical miles; but in the chart we have doubled it and made it equal in length to sixty nautical miles at the equator; therefore to preserve the proportion we make the distance between the parallels of 59° 30′ and 60° 30′, equal in length to 120 nautical miles at the equator. (Of course, the distance on the chart between 59° 30′ and 59° 31′ is less than the distance between 59° 31′ and 59° 32′, and so on). This length, however, is only divided into sixty miles; and distances on the chart are always measured at the right or left hand side, the upper part of the chart being north, and not at the top or bottom of the chart, because its size is exaggerated in all directions in the same proportion.

The *meridional parts* (Inman, table 7; Norie, table III.) of a certain latitude give the length expressed in nautical miles of the line on the Mercator's chart which represents the latitude, or, in other words, the length of the meridian on a Mercator's chart intercepted between the equator and the parallel of latitude measured in nautical miles at the equator on such a chart.

The *meridional difference of latitude* (mer. diff. lat.)

between two places is the difference between the meridional parts of their respective latitudes taken from the tables mentioned in the last paragraph.

Plane sailing is the method of solving problems in the case of a ship sailing on a rhumb line by the use of plane trigonometry, and the relations existing between the course, distance, departure, and difference of latitude.

Traverse sailing deals with the position of a ship which has sailed on several courses.

Mercator's sailing is the method of solving problems, when the course is a rhumb line, by the use of the Mercator's chart and the table of meridional parts.

Parallel sailing enables us to find the position of a ship after she has sailed a due east or west course, or, in other words, when her course has been on a parallel of latitude.

Middle latitude sailing is the method of using the departure as though it were all sailed on the middle latitude between the place from which the ship started and that at which she arrived.

Let A and B (Fig. 6) be two places on the surface of the earth. The meridian of A is P N P_1, and the meridian of B is P T P_1. Let X be the position of Greenwich, then P M P_1 is the first or prime meridian. The longitude of A is M N, and the longitude of B is M T. The latitude of A is N A, and the latitude of B is T B. The difference of latitude between A and B is S A or R B, and the difference of longitude is N T.

THE CONSTRUCTION OF ASTRONOMICAL DIAGRAMS.

We know that the sun, moon, planets, and stars which appear to us to be dotted about on the celestial concave really differ immensely in their distances from the earth. But for the purpose of solving problems in nautical astronomy we must imagine that they are all placed on the inside of an enormous sphere, called the celestial sphere. Let us now extend the axis of the earth to this sphere, and call its ends the *celestial poles;* the terrestrial equator, similarly extended, is named the *celestial equator* or equinoctial; the terrestrial meridians, *celestial meridians;* the parallels of latitude extended become *parallels of declination;* the point immediately above the observer's head we call the *zenith,* and that in the celestial concave directly under him the *nadir;* a great circle 90° from the zenith and nadir, the *rational horizon;* great circles passing through the zenith and nadir, and cutting the horizon at right angles, *circles of altitude;* a great circle passing through the zenith and nadir and through the east and west points of the horizon, the *prime vertical;* the meridian passing through the zenith and nadir, the *celestial meridian of the observer.*

Let Fig. 7 be a representation of the celestial globe. We must imagine ourselves to be at an immense height immediately above some part of the celestial equator looking down upon this globe. It would appear to us as in the figure; P, P^1 being the poles, W Q E, the equator; P A P^1, P Q P^1, &c., meridians of the celestial globe. Now the position of a heavenly body is determined by its right ascension and declination in exactly the same manner as the position of a place on the earth by its latitude and longitude.

The declination of a heavenly body corresponds in every way with the latitude of a terrestrial body, and is the arc of *a celestial meridian intercepted between the celestial equator and the centre of such body.* Thus, if Y and X are two heavenly bodies, A Y and S X are their respective declinations. The right ascension of a heavenly body corresponds with the longitude of a place on the earth, but is measured eastward from the first point of Aries instead of being measured, as longitude is, eastward and westward from the meridian of Greenwich. To understand what is the first point of Aries, we must first of all know what is the ecliptic. As the earth revolves round the sun, she bows first one pole and then the other towards him, because her axis is inclined to her orbit at an angle of about 66° 32′ 15″. The sun, therefore, appears to describe a

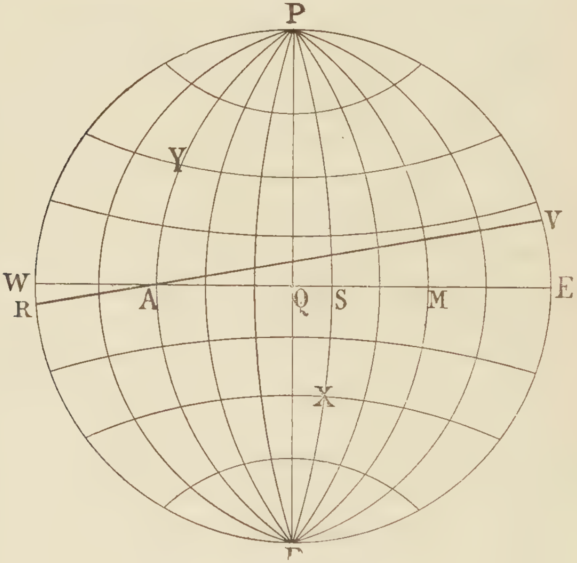

FIG. 7.

path in the heavens which twice in the year crosses the equinoctial, on about March 25 and Sept. 25; these are called the vernal and autumnal equinoxes, because the length of the day and night at such times is equal all over the world. The point of intersection of the equator and the ecliptic at the vernal equinox is called the first point of Aries. In Fig. 7 R A V is the ecliptic, and the point A is the first point of Aries.

The right ascension of a heavenly body is the arc of the celestial equator intercepted between the first point of Aries and the meridian passing through the centre of such body measured eastward from the first point of Aries. Thus in Fig. 7 the right ascension of Y is zero, because it is situated on the meridian which passes through the first point of Aries, and the right ascension of X is A S. (The right ascension is always measured in time, 24 hours being equal to 360°.)

Fig. 7 is said to be on the plane of the meridian P W P_1E.

Fig. 8 is constructed on the plane of the horizon N W S E, and in this case we are supposed to be looking at the celestial globe from a point directly above the zenith of the observer on the earth; this zenith is marked in the figure Z. N Z S is the meridian of the observer; W Z E the prime vertical. Now the latitude of an observer on the surface of the earth is the arc of the terrestrial meridian intercepted between the equator and himself, and this, as we know, is measured by the angle at the centre of the earth; but an arc of the celestial meridian is similarly measured, and therefore the angular value of the arc measuring the latitude on the terrestrial meridian is equal to a corresponding arc measured on the celestial meridian. Let Z Q be such an arc corresponding to the arc of the terrestrial meridian which is the latitude; then Z Q must equal the latitude of the observer, and W Q E be part of the equator. Now the whole length N S being half the circumference of the celestial globe is equal to 180°.

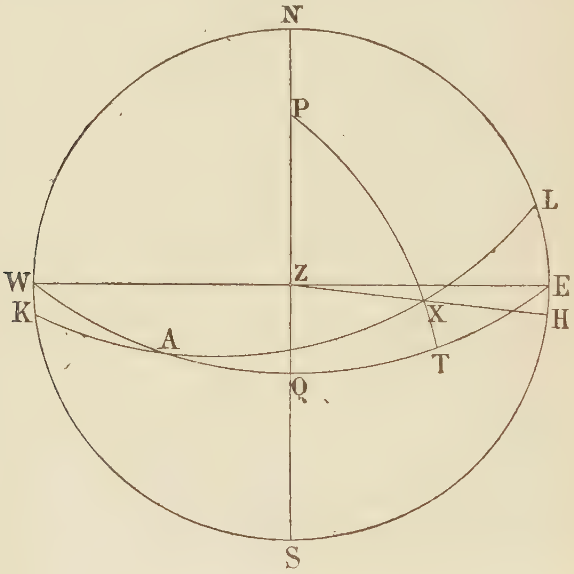

FIG. 8.

Make Q P equal to 90°. Then P is the north celestial pole, which is 90° from every part of the equator W Q E. Let X be the position of the sun, and P T part of the meridian passing through his centre. Then, as we have seen above, X T is his declination north of the equator, and T A (A being the first point of Aries) his right ascension. Now Z H is part of a circle of altitude (*see* definition above) passing through the sun; H X, the arc of the circle of altitude intercepted between the horizon and the sun's centre is his *altitude;* Z X, the complement of X H, since Z H is equal to 90°, is the sun's *zenith distance;* P X is the sun's distance from the pole, and is called the sun's *polar distance,* always measured from the elevated pole, that is, from the pole which is above the horizon of the observer.

Now P Z X is a spherical triangle; the sides P Z, P X, and Z X being arcs of great circles. P Z = P Q − Z Q = 90° − latitude = co-latitude of observer. Z X = 90° − X H = 90° − altitude = zenith distance. And P X = 90° − X T = 90° − declination = polar distance: if the sun is situated to the southward of the equator in north latitudes, or to the northward of the equator in south latitudes, it is evident that his declination must be added to 90° to find his polar distance. Having three parts of any spherical triangle we are able to find all the other parts. We can find, therefore, the angle Z P X, which is called the hour angle of the sun, and is used in solving chronometer observations and other problems connected with time; also P Z X, or the sun's bearing or azimuth, which is of the first importance in finding the compass error.

CHAPTER VIII.

SPHERICAL TRIGONOMETRY.

UPON spherical trigonometry depends the solution of all problems in nautical astronomy.

A spherical triangle is a three-sided figure drawn on the surface of a sphere, whose sides are arcs of great circles, and whose angles are those formed by the planes of such great circles.

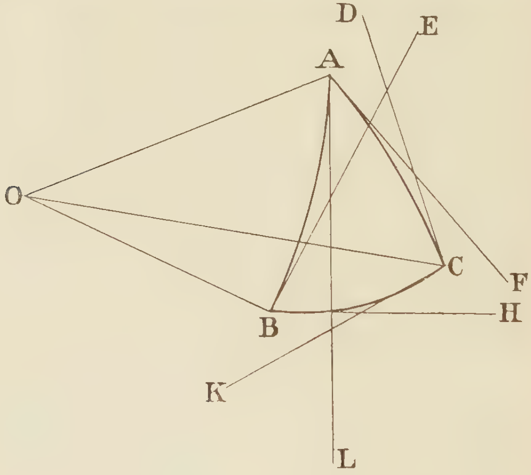

FIG. 9.

Let O be the centre of a sphere, upon the surface of which is described the triangle A B C. The sides A B, B C, and A C being arcs of great circles, the triangle A B C is a spherical triangle whose sides, A B, B C, and A C, are measured by the angles A O B, B O C, and A O C respectively, and whose angles are the inclinations of the planes of these arcs. Thus, the angle at A is the angle of inclination between the planes A O B and A O C; or, if we draw tangents from A in the same planes as A B and A C, it is the angle F A L. Similarly, the angle at B is E B H, and the angle at C, D C K.

It may be well here to mention the principal properties of spherical triangles.

Each side of a spherical triangle is less than a semicircle.

Each angle of a spherical triangle is less than two right angles.

Any two sides of a spherical triangle are together greater than the third.

If two angles of a spherical triangle are equal, the sides opposite to them are equal.

If two sides of a spherical triangle are equal, the angles opposite to them are equal.

The greater angle in a spherical triangle always subtends the greater side.

The three angles of a spherical triangle are greater than two right angles, and less than six right angles.

The three sides of a spherical triangle are together less than four right angles.

The solution of spherical triangles is greatly dependent upon a knowledge of Euclid. I shall, however, in the following investigations merely mention the proposition involved, so that my readers may themselves refer to them.

It is impossible in this work to give the proofs *in extenso.* My object in presenting the following proof of the formulæ used in the chronometer problem is to enable my readers to comprehend the method by which these formulæ have been evolved. It is, I venture to think, a most interesting study. I hope that this short exposition of analytical trigonometry may induce some of my readers to take up the subject more extensively.

FORMULÆ TO BE PROVED.

(*a*) $\text{Sin.}\,^2\text{A} + \text{Cos.}\,^2\text{A} = 1.$

(*b*) $\text{Cos. A} = \dfrac{b^2 + c^2 - a^2}{2\,b\,c}$ in any plane triangle.

(*c*) $\text{Sin. (A + B)} = \text{Sin. A . Cos. B} + \text{Cos. A . Sin. B.}$

(*d*) $\text{Cos. (A + B)} = \text{Cos. A . Cos. B} - \text{Sin. A . Sin. B.}$

(*e*) $\text{Sin. (A − B)} = \text{Sin. A . Cos. B} - \text{Cos. A . Sin. B.}$

(*f*) $\text{Cos. (A − B)} = \text{Cos. A . Cos. B} + \text{Sin. A . Sin. B.}$

(*g*) $\text{Sin. A} + \text{Sin. B} = 2\ \text{Sin.}\,\frac{1}{2}\,(\text{A} + \text{B})\,.\,\text{Cos.}\,\frac{1}{2}\,(\text{A} - \text{B}).$

(*h*) $\text{Sin. A} - \text{Sin. B} = 2\ \text{Cos.}\,\frac{1}{2}\,(\text{A} + \text{B})\,.\,\text{Sin.}\,\frac{1}{2}\,(\text{A} - \text{B}).$

(*k*) $\text{Cos. A} + \text{Cos. B} = 2\ \text{Cos.}\,\frac{1}{2}\,(\text{A} + \text{B})\,.\,\text{Cos.}\,\frac{1}{2}\,(\text{A} - \text{B}).$

(*l*) $\text{Cos. B} - \text{Cos. A} = 2\ \text{Sin.}\,\frac{1}{2}\,(\text{A} + \text{B})\,.\,\text{Sin.}\,\frac{1}{2}\,(\text{A} - \text{B}).$

(*m*) $\text{Sin. 2 A} = 2\ \text{Sin. A . Cos. A.}$

(*n*) $\text{Cos. 2 A} = \text{Cos.}\,^2\text{A} - \text{Sin.}\,^2\text{A} = 1 - 2\ \text{Sin.}\,^2\text{A} = 2\ \text{Cos.}\,^2\text{A} - 1.$

(*o*) $\text{Cos. A} = \dfrac{\text{Cos.}\,a - \text{Cos.}\,b\,.\,\text{Cos.}\,c}{\text{Sin.}\,b\,.\,\text{Sin.}\,c}$ in any spherical triangle.

(*p*) $\text{Sin.}\,\dfrac{^2\text{A}}{2} = \dfrac{\text{Sin.}\,\frac{1}{2}\,(a + b - c)\,.\,\text{Sin.}\,\frac{1}{2}\,(a - b + c)}{\text{Sin.}\,b\,.\,\text{Sin.}\,c}$ in any spherical triangle.

(*r*) $\text{Sin.}\,\dfrac{\text{A}}{2} = \sqrt{\dfrac{\text{Sin.}\,\frac{1}{2}\,(a + b - c)\,.\,\text{Sin.}\,\frac{1}{2}\,(a - b + c)}{\text{Sin.}\,b\,.\,\text{Sin.}\,c}}$ in any spherical triangle.

(s) $\text{Hav } A = \sqrt{\text{Hav } (a + b - c)} \cdot \sqrt{\text{Hav } (a - b + c)}$ Cosec b . Cosec c . in any spherical triangle.

Let A B C be a plane triangle, draw B D at right angles to A C.

A B is the side c.
B C „ „ a.
A C „ „ b.

(a) To prove that $\text{Sin.}\,^2A + \text{Cos.}\,^2A = 1$.
In Fig. 10.

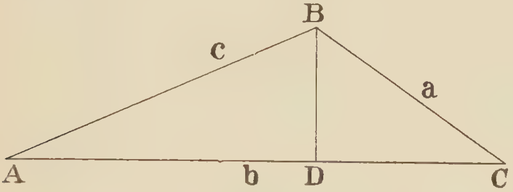

Fig. 10.

In the right angled triangle A B D

$$\text{Sin. } A = \frac{DB}{AB} \therefore \text{Sin.}\,^2A = \frac{DB^2}{AB^2}$$

$$\text{Cos. } A = \frac{AD}{AB} \therefore \text{Cos.}\,^2A = \frac{AD^2}{AB^2}$$

$$\therefore \text{Sin.}\,^2A + \text{Cos.}\,^2A = \frac{DB^2}{AB^2} + \frac{AD^2}{AB^2} = \frac{DB^2 + AD^2}{AB^2}$$

But A D B is a right angled triangle, and therefore the square on the hypothenuse is equal to the sum of the squares on the other two sides (Euclid I., 47).

$$\therefore DB^2 + AD^2 = AB^2$$

$$\text{and Sin.}\,^2A + \text{Cos.}\,^2A = \frac{DB^2 + AB^2}{AB^2} = \frac{AB^2}{AB^2} = 1$$

$$\text{also Sin.}\,^2A = 1 - \text{Cos.}\,^2A$$

$$\text{and Cos.}\,^2A = 1 - \text{Sin.}\,^2A.$$

(*b*) To prove that Cos. A $= \frac{b^2 + c^2 - a^2}{2\,b\,c}$

In any plane triangle "the square on the side subtending an acute angle is less than the squares on the sides containing that angle by twice the rectangle contained by either of these sides, and the straight line intercepted between the perpendicular let fall on it from the opposite angle and the acute angle" (Euclid II., 13).

Now in the plane triangle A B C (Fig. 10) B D is perpendicular to A C.

$$\therefore \text{B C}^2 = \text{A B}^2 + \text{A C}^2 - 2\,\text{A C. A D}$$

$$\text{Now Cos. A} = \frac{\text{A D}}{\text{A B}} \ \therefore \ \text{A D} = \text{A B. Cos. A},$$

and substituting this expression for A D we have

$$\text{B C}^2 = \text{A B}^2 + \text{A C}^2 - 2\,\text{A C . A B . Cos. A}$$

$$\text{or } a^2 = b^2 + c^2 - 2\,b.\,c.\ \text{Cos. A}$$

$$2\,b\,c\ \text{Cos. A} = b^2 + c^2 - a^2$$

$$\text{or Cos. A} = \frac{b^2 + c^2 - a^2}{2\,b.\,c.}$$

(*c*) To prove that Sin. (A + B) = Sin. A . Cos. B + Cos. A . Sin. B.

(*d*) and Cos. (A + B) = Cos. A . Cos. B − Sin. A . Sin. B.

Let the angle B R C be equal to A, and the angle C R D be equal to B. The angle B R D will, therefore, be equal to A + B.

Draw O N perpendicular to R C, N P perpendicular to R B, O Q perpendicular to R B, and N M perpendicular to O Q. Then the angle M O N = 90° − M N O = M N R = A, because M N is parallel to R B (Euclid I., 29).

$$\text{Now Sin. (A + B)} = \frac{\text{O Q}}{\text{R O}} = \frac{\text{O M + N P}}{\text{O R}} = \frac{\text{O M}}{\text{O R}} + \frac{\text{N P}}{\text{O R}}$$

Multiply the numerator and denominator of the first fraction by O N, and those of the second by R N (which, of course, does not alter the values of these fractions) and we have:

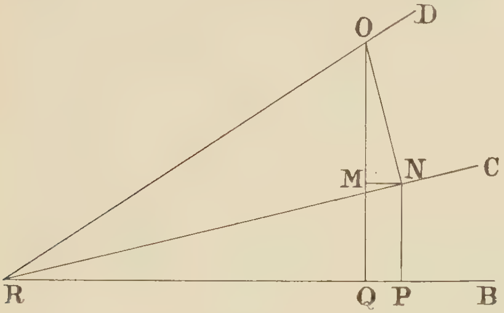

Fig. 11.

$$\text{Sin. } (A + B) = \frac{OM}{OR} \times \frac{ON}{ON} + \frac{NP}{OR} \times \frac{RN}{RN}$$

$$\text{or } \frac{OM}{ON} \times \frac{ON}{OR} + \frac{NP}{RN} \times \frac{RN}{OR}$$

$$= \text{Cos. } MON \times \text{Sin. } ORN + \text{Sin. } CRB \times \text{Cos. } DRC$$

$$= \text{Cos. } A \times \text{Sin } B + \text{Sin. } A \times \text{Cos. } B.$$

$$\text{and Cos. } (A + B) = \frac{RQ}{RO} = \frac{RP - PQ}{RO} = \frac{RP - MN}{RO}$$

$$= \frac{RP}{RN} \times \frac{RN}{RO} - \frac{MN}{ON} \times \frac{ON}{RO}$$

$$= \text{Cos. } A \times \text{Cos. } B - \text{Sin. } A \times \text{Sin. } B.$$

(*e*) To prove that Sin. (A − B) = Sin. A . Cos. B − Cos. A . Sin. B.

(*f*) and Cos. (A − B) = Cos. A . Cos. B + Sin. A . Sin. B.

Let the angle B R D be equal to A, and the angle D R C be equal to B, then the angle B R C will be equal to A−B. Draw O N perpendicular to R D, Q O, and N P perpendicular to R B, and O M perpendicular to P N. projected to M.

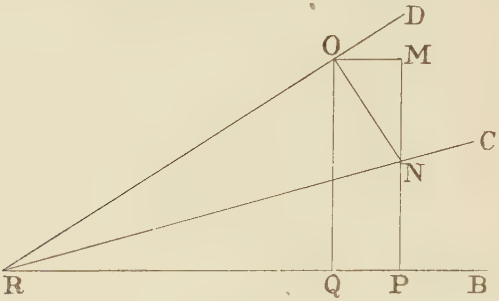

FIG. 12.

The angle $O N M = 90^0 - N O M = D O M = A$ because $O M$ is parallel to $R B$

$$\text{Now Sin.}\,(A-B) = \frac{NP}{RN} = \frac{OQ - NM}{RN} = \frac{OQ}{RN} - \frac{NM}{RN}$$

$$= \frac{OQ}{RO} \times \frac{RO}{RN} - \frac{NM}{ON} \times \frac{ON}{RN}$$

$$= \text{Sin. A. Cos. B} - \text{Cos. A. Sin. B}$$

$$\text{And Cos.}\,(A-B) = \frac{AP}{AN} = \frac{AQ + OM}{AN} = \frac{AQ}{AN} = \frac{OM}{AN}$$

$$= \frac{AQ}{AO} \times \frac{AO}{AN} + \frac{OM}{ON} \times \frac{ON}{AN}$$

$$= \text{Cos. A. Cos. B} + \text{Sin. A. Sin. B.}$$

To find values for Sin. A + Sin. B, Sin. A − Sin. B, Cos. A + Cos. B, and Cos. B − Cos. A.

$$\text{Sin. A} = \text{Sin.}\left\{\tfrac{1}{2}(A+B) + \tfrac{1}{2}(A-B)\right\} \text{ because } \tfrac{1}{2}(A+B) + \tfrac{1}{2}(A-B) = A.$$

$$= \text{Sin. } \tfrac{1}{2}(A+B)\,.\,\text{Cos. } \tfrac{1}{2}(A-B) + \text{Cos. } \tfrac{1}{2}(A+B)\,.\,\text{Sin. } \tfrac{1}{2}(A-B) \text{ (see } c\text{).}$$

$$\text{Sin. B} = \text{Sin.} \left\{ \tfrac{1}{2}(\text{A}+\text{B}) - \tfrac{1}{2}(\text{A}-\text{B}) \right\}$$
$$= \text{Sin.}\ \tfrac{1}{2}(\text{A}+\text{B}) \,.\, \text{Cos.}\ \tfrac{1}{2}(\text{A}-\text{B}) - \text{Cos.}\ \tfrac{1}{2}(\text{A}+\text{B}) \,.\, \text{Sin.}\ \tfrac{1}{2}(\text{A}-\text{B}).$$

$$\text{Cos. A} = \text{Cos.} \left\{ \tfrac{1}{2}(\text{A}+\text{B}) + \tfrac{1}{2}(\text{A}-\text{B}) \right\}$$
$$= \text{Cos.}\ \tfrac{1}{2}(\text{A}+\text{B}) \,.\, \text{Cos.}\ \tfrac{1}{2}(\text{A}-\text{B}) - \text{Sin.}\ \tfrac{1}{2}(\text{A}+\text{B}) \,.\, \text{Sin.}\ \tfrac{1}{2}(\text{A}-\text{B}).$$

$$\text{Cos. B} = \text{Cos.} \left\{ \tfrac{1}{2}(\text{A}+\text{B}) - \tfrac{1}{2}(\text{A}-\text{B}) \right\}$$
$$= \text{Cos.}\ \tfrac{1}{2}(\text{A}+\text{B}) \,.\, \text{Cos,}\ \tfrac{1}{2}(\text{A}-\text{B}) + \text{Sin.}\ \tfrac{1}{2}(\text{A}+\text{B}) \,.\, \text{Sin.}\ \tfrac{1}{2}(\text{A}-\text{B}).$$

Adding and subtracting these four expressions in pairs we obtain the following results:

(*g*) $\text{Sin. A} + \text{Sin. B} = 2\ \text{Sin.}\ \tfrac{1}{2}(\text{A}+\text{B}) \,.\, \text{Cos.}\ \tfrac{1}{2}(\text{A}-\text{B}).$

(*h*) $\text{Sin. A} - \text{Sin. B} = 2\ \text{Cos.}\ \tfrac{1}{2}(\text{A}+\text{B}) \,.\, \text{Sin.}\ \tfrac{1}{2}(\text{A}-\text{B}).$

(*k*) $\text{Cos. A} + \text{Cos. B} = 2\ \text{Cos.}\ \tfrac{1}{2}(\text{A}+\text{B}) \,.\, \text{Cos.}\ \tfrac{1}{2}(\text{A}-\text{B}).$

(*l*) $\text{Cos. B} - \text{Cos. A} = 2\ \text{Sin.}\ \tfrac{1}{2}(\text{A}+\text{B}) \,.\, \text{Sin.}\ \tfrac{1}{2}(\text{A}-\text{B}).$

For example:

$$\text{Sin. A} = \text{Sin.}\ \tfrac{1}{2}(\text{A}+\text{B}) \,.\, \text{Cos.}\ \tfrac{1}{2}(\text{A}-\text{B}) + \text{Cos.}\ \tfrac{1}{2}(\text{A}+\text{B}) \,.\, \text{Sin.}\ \tfrac{1}{2}(\text{A}-\text{B}).$$

$$\text{Sin. B} = \text{Sin.}\ \tfrac{1}{2}(\text{A}+\text{B}) \,.\, \text{Cos.}\ \tfrac{1}{2}(\text{A}-\text{B}) - \text{Cos.}\ \tfrac{1}{2}(\text{A}+\text{B}) \,.\, \text{Sin.}\ \tfrac{1}{2}(\text{A}-\text{B}).$$

And adding them together we have the result given above, namely:

$$\text{Sin. A} + \text{Sin. B} = 2\ \text{Sin.}\ \tfrac{1}{2}(\text{A}+\text{B}) \,.\, \text{Cos.}\ \tfrac{1}{2}(\text{A}-\text{B}).$$

If in the formula $\text{Sin.}\ (\text{A}+\text{B}) = \text{Sin. A} \,.\, \text{Cos. B} + \text{Cos. A} \,.\, \text{Sin. B}$ we make A equal to B, we obtain:

(*m*) $$\text{Sin. 2 A} = \text{Sin.}\ (\text{A}+\text{A}) = \text{Sin. A} \,.\, \text{Cos. A} + \text{Cos. A} \,.\, \text{Sin. A}.$$
$$= 2\ \text{Sin. A} \,.\, \text{Cos. A}.$$

(*n*) Also $\text{Cos. } 2\text{ A} = \text{Cos. } (\text{A} + \text{A}) = \text{Cos. A.Cos. A} - \text{Sin. A.Sin. A.}$

$$= \text{Cos. }^2\text{A} - \text{Sin. }^2\text{A.}$$

And since $\text{Cos. }^2\text{A} = 1 - \text{Sin. }^2\text{A.}$

$$\therefore \text{Cos. } 2\text{ A} = 1 - 2\text{ Sin. }^2\text{A.}$$

And that $\text{Sin. }^2\text{A} = 1 - \text{Cos. }^2\text{A.}$

$$\therefore \text{Cos. } 2\text{ A} = \text{Cos. }^2\text{A} - (1 - \text{Cos. }^2\text{A}$$

$$= \text{Cos. }^2\text{A} - 1 + \text{Cos. }^2\text{A.}$$

$$= 2\text{ Cos. }^2\text{A} - 1.$$

Substituting A for 2 A

$$\text{Cos. A} = 1 - 2\text{ Sin.}^2\,\frac{\text{A}}{2}.$$

$$= 2\text{ Cos.}^2\,\frac{\text{A}}{2} - 1.$$

(*o*) To prove that in a spherical triangle—

$$\text{Cos. A} = \frac{\text{Cos. } a - \text{Cos. } b,\ \text{Cos. } c}{\text{Sin. } b,\ \text{Sin. } c}$$

Let A B C be a spherical triangle, O the centre of the sphere; let the tangent at A to the arc A B meet O B produced in D; and let the tangent at A to the arc A C meet O C produced in E; join D E.

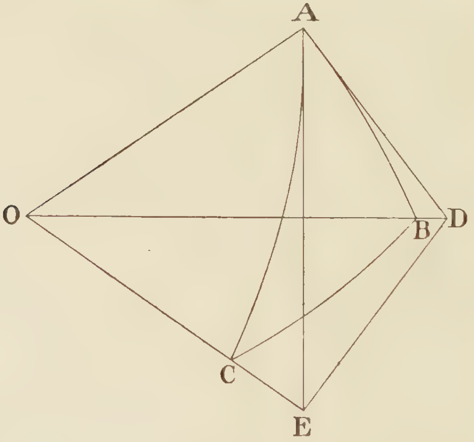

Fig. 13.

The angle D A E measures the angle A of the triangle, and the angle D O E measures the side C B, or a.

In the plane triangle A D E

$$DE^2 = AD^2 + AE^2 - 2\,AD\,.\,AE\,.\,\text{Cos. }DAE \text{ (see } b\text{)}.$$

And in the plane triangle O D E

$$DE^2 = OD^2 + OE^2 - 2\,OD\,.\,OE\,.\,\text{Cos. }DOE \text{ (see } b\text{)}.$$

But angle D A E = angle A in the spherical triangle A B C

And angle D O E = side a in the spherical triangle A B C

$$\therefore DE^2 = AD^2 + AE^2 - 2\,AD\,.\,AE\,.\,\text{Cos. A}$$
$$\text{and } DE^2 = DO^2 + EO^2 - 2\,DO\,.\,EO\,.\,\text{Cos. } a.$$

Now subtracting each side of the first equation from the second we have—

$$O = DO^2 - AD^2 + EO^2 - EA^2 + 2\,AD\,.\,AE\,.\,\text{Cos. A} - 2\,DO\,.\,EO\,.\,\text{Cos. } a.$$

The angles O A D and O A E are right angles, A D and A E being tangents to the sphere and therefore perpendicular to the radius O A, and the triangles O A D and O A E are right angled plane triangles.

$$\therefore OE^2 = OA^2 + AE^2, \text{ and } OD^2 = OA^2 + AD^2 \text{ (Euclid I., 47)},$$
$$\text{and } OA^2 = OE^2 - AE^2 = OD^2 - AD^2.$$

Now substituting OA^2 for these expressions in the equation we have

$$O = 2\,AO^2 + 2\,AD\,.\,AE\,.\,\text{Cos. A} - 2\,DO\,.\,EO\,.\,\text{Cos. } a$$
$$\text{and } 2\,DO\,.\,OE\,.\,\text{Cos. } a = 2\,AO^2 + 2\,AD\,.\,AE\,.\,\text{Cos. A}$$

or dividing all the terms by 2

$$DO\,.\,OE\,.\,\text{Cos. } a = AO^2 + AD\,.\,AE\,.\,\text{Cos. A}$$
$$\text{Cos. } a = \frac{AO^2 + AD\,.\,AE\,.\,\text{Cos. A}}{OE\,.\,OD}$$

$$=\frac{AO\,.\,AO}{OE\,.\,OD}+\frac{AE\,.\,AD}{OE\,.\,OD}\,.\,\text{Cos. A}$$

$$=\frac{AO}{OE}\,.\,\frac{AO}{OD}+\frac{AE}{OE}\,.\,\frac{AD}{OD}\,.\,\text{Cos. A}$$

Now $\frac{OA}{OE}=$ Cos. A O E because A O E is a right angled triangle.

$\frac{AO}{OD}=$ Cos. A O D because O A D is a right angled triangle.

$\frac{AE}{OE}=$ Sin. A O E and $\frac{AD}{OD}=$ Sin. A O D.

The angles A O E and A O D measure respectively the arcs A C and A B, which are the sides b and c of the spherical triangle A B C. Substituting we have then

$$\text{Cos. } a=\text{Cos. } b\,.\,\text{Cos. } c+\text{Sin. } b\,.\,\text{Sin. } c\,.\,\text{Cos. A.}$$

$$\therefore \text{Cos. A}\,.\,\text{Sin. } b\,.\,\text{Sin. } c=\text{Cos. } a-\text{Cos. } b\,.\,\text{Cos. } c.$$

(*o*) $$\text{and Cos. A}=\frac{\text{Cos. } a-\text{Cos. } b\,.\,\text{Cos. } c}{\text{Sin. } b\,.\,\text{Sin. } c.}$$

Similarly it can be proved that—

$$\text{Cos. B}=\frac{\text{Cos. } b-\text{Cos. } a\,.\,\text{Cos. } c}{\text{Sin. } a\,.\,\text{Sin. } c}$$

and $$\text{Cos. C}=\frac{\text{Cos. } c-\text{Cos. } a\,.\,\text{Cos. } b}{\text{Sin. } a\,.\,\text{Sin. } b.}$$

(*p*) (*r*) (*s*) To express the sine of half an angle of a spherical triangle in terms of the sides. Since in spherical triangle A B C (p. 66).

$$\text{Cos. A}=\frac{\text{Cos. } a-\text{Cos. } b\,.\,\text{Cos. } c}{\text{Sin. } b\,.\,\text{Sin. } c.}$$ (See *o*)

$$\text{therefore } 1-\text{Cos. A}=1-\frac{\text{Cos. } a-\text{Cos. } b\,.\,\text{Cos. } c}{\text{Sin. } b\,.\,\text{Sin. } c}$$

$$= \frac{\text{Sin. } b \,.\, \text{Sin. } c - (\text{Cos. } a - \text{Cos. } b \,.\, \text{Cos. } c)}{\text{Sin. } b \,.\, \text{Sin. } c}$$

$$= \frac{\text{Sin. } b \,.\, \text{Sin. } c - \text{Cos. } a + \text{Cos. } b \,.\, \text{Cos. } c}{\text{Sin. } b \,.\, \text{Sin. } c.}$$

but $\text{Sin. } b \,.\, \text{Sin. } c + \text{Cos. } b \,.\, \text{Cos. } c = \text{Cos. } (b - c)$ (See f)

$$\therefore 1 - \text{Cos. A} = \frac{\text{Cos. } (b - c) - \text{Cos. } a}{\text{Sin. } b \,.\, \text{Sin. } c}$$

but $\text{Cos. A} = 1 - 2 \text{ Sin.}^2 \frac{\text{A}}{2}$ (See n)

$$\therefore 1 - \text{Cos. A} = 1 - (1 - 2 \text{ Sin.}^2 \frac{\text{A}}{2})$$

$$= 1 - 1 + 2 \text{ Sin.}^2 \frac{\text{A}}{2}$$

$$= 2 \text{ Sin.}^2 \frac{\text{A}}{2}$$

Now $\text{Cos. B} - \text{Cos. A} = 2 \text{ Sin. } \tfrac{1}{2} (\text{A} + \text{B}).\ \text{Sin. } \tfrac{1}{2} (\text{A} - \text{B})$ (See l)

$$\therefore \text{Cos. } (b - c) - \text{Cos. } a = 2 \text{ Sin. } \tfrac{1}{2} (a + b - c).\ \text{Sin. } \tfrac{1}{2} \left\{ (a - (b - c) \right\}$$

$$= 2 \text{ Sin. } \tfrac{1}{2} (a + b - c).\ \text{Sin. } \tfrac{1}{2} (a - b + c)$$

therefore, substituting these values in the equation above we have

$$2 \text{ Sin.}^2 \frac{\text{A}}{2} = \frac{2 \text{ Sin. } \frac{1}{2} (a + b - c).\ \text{Sin. } \frac{1}{2} (a - b + c)}{\text{Sin. } b \,.\, \text{Sin. } c}$$

$$(p) \text{ or Sin.}^2 \frac{\text{A}}{2} = \frac{\text{Sin. } \frac{1}{2} (a + b - c).\ \text{Sin. } \frac{1}{2} (a - b + c)}{\text{Sin. } b \,.\, \text{Sin. } c}$$

$$(r) \text{ Sin.} \frac{\text{A}}{2} = \sqrt{\frac{\text{Sin. } \frac{1}{2} (a + b - c).\ \text{Sin. } \frac{1}{2} (a - b + c)}{\text{Sin. } b \,.\, \text{Sin. } c.}}$$

but $\text{Sin.}^2 \frac{\text{A}}{2}$ = Haversine A = Horary Angle (Norie)

for $1 - \text{Cos. A}$ = Versine A = $2 \text{ Sin.}^2 \frac{\text{A}}{2}$

$\therefore$ Half Versine: or Haversine A = $\text{Sin.}^2 \frac{\text{A}}{2}$

To express the haversine of an angle of a spherical triangle in terms of the sides. As we have already seen,

$$1 - \text{Cos. A} = 1 - \frac{\text{Cos. } a - \text{Cos. } b \,.\, \text{Cos. } c}{\text{Sin. } b \,.\, \text{Sin. } c.}$$

$$\therefore \text{Vers. A} = \frac{\text{Sin. } b \,.\, \text{Sin. } c - \text{Cos. } a + \text{Cos. } b \,.\, \text{Cos. } c.}{\text{Sin. } b \,.\, \text{Sin. } c}$$

$$\text{and } 2 \text{ Hav. A} = \frac{\text{Cos. } (b - c) - \text{Cos. } a}{\text{Sin. } b \,.\, \text{Sin. } c}$$

$$= \frac{2 \text{ Sin. } \tfrac{1}{2} (a + b - c).\ \text{Sin. } \tfrac{1}{2} (a - b + c)}{\text{Sin. b . Sin. } c}$$

$$\text{therefore Hav. A} = \frac{\text{Sin. } \tfrac{1}{2} (a + b - c) \,.\, \text{Sin. } \tfrac{1}{2} (a - b + c}{\text{Sin. } b \,.\, \text{Sin. } c}$$

$$\text{but Sin.}^2 \frac{\text{A}}{2} = \text{Haversine A.}$$

$$\text{and Sin. } \tfrac{1}{2} (a + b - c) = \sqrt{\text{Hav. } (a + b - c)}$$

$$\text{Sin. } \tfrac{1}{2} (a - b + c) = \sqrt{\text{Hav. } (a - b + c)}$$

$$\text{therefore Hav. A} = \frac{\sqrt{\text{Hav.} (a + b - c)} \,.\, \sqrt{\text{Hav.} (a - b + c)}}{\text{Sin. } b \,.\, \text{Sin. } c.}$$

(s) $\text{And Hav. A} = \sqrt{\text{Hav. } (a + b - c)} \,.\, \sqrt{\text{Hav. } (a - b + c)} \,.\, \text{Cosec. } b \,.\, \text{Cosec. } c.$

It will easily be seen by examination of the trigonometrical ratios in Chapter V. that

$$\text{Sin. A} = \frac{1}{\text{Cosec. A}} \text{ or Cosec. A} = \frac{1}{\text{Sin. A}}.$$

Norie's rule for finding the horary or hour angle is derived from formula (p), thus :

In the spherical triangle P X Z, let P be the Pole, Z the zenith, and X the position of a heavenly body. P Z is a portion of the meridian equal to 90° − latitude, P X is the polar distance of X which is equal to 90° ±* declination, Z X is its zenith distance which is equal to 90° — altitude, the angle Z P X is the horary angle, and the angle P Z X is the azimuth. We will represent the horary angle by H,

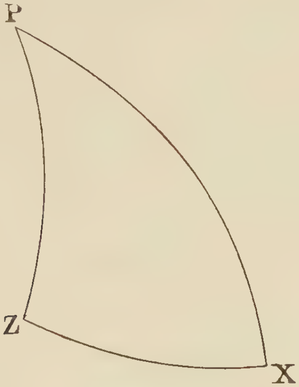

FIG. 14.

the azimuth by A, the latitude by l, the declination by d, the altitude by a, the polar distance by p, and the zenith distance by Z.

$$\therefore\ PZ \text{ the co-latitude} = 90^\circ - l$$
$$PX = p \text{ and } ZX = 90^\circ - a.$$

And by formula (p)

$$\text{Sin.}^2 \frac{H}{2} = \frac{\text{Sin.}\,\frac{1}{2}\,(ZX + PZ - PX)\,.\,\text{Sin.}\,\frac{1}{2}\,(ZX - PZ + PX)}{\text{Sin. } PZ\,.\,\text{Sin. } PX}$$

$$= \frac{\text{Sin.}\,\frac{1}{2}\left\{(90^\circ - a) + (90 - l) - p\right\}.\,\text{Sin.}\,\frac{1}{2}\left\{(90^\circ - a) - (90^\circ - l) + p\right\}}{\text{Sin. }(90^\circ - l)\,.\,\text{Sin. } p.}$$

---

* The sign ± means that the quantity may be added to or subtracted from the quantity preceding it according to circumstances. In this case the declination is to be added to 90° if it is of a different name to the latitude, but to be subtracted from it if it is of the same name.

$$\text{Now, } \tfrac{1}{2}\left\{(90 - a) + (90 - l) - p\right\} = \tfrac{1}{2}\,(180° - (a + p + l)$$
$$= 90° - \tfrac{1}{2}\,(a + p + l)$$

$$\text{And Sin.}\left\{90° - \tfrac{1}{2}\,(a + p + l)\right\} = \text{Cos. } \tfrac{1}{2}\,(a + p + l)$$

$$\text{Again, } \tfrac{1}{2}\left\{(90° - a) - (90° - l) + p = \tfrac{1}{2}\,(90° - a - 90° + l + p)\right.$$
$$= \tfrac{1}{2}\,(p + l - a)$$
$$= \tfrac{1}{2}\,p + \tfrac{1}{2}\,l + \tfrac{1}{2}\,a - a$$
$$= \tfrac{1}{2}\,(p + l + a) - a.$$

$$\text{And } \frac{1}{\text{Sin. }(90 - l)} = \frac{1}{\text{Cos. } l} = \text{Secant. } l.$$

$$\text{Also } \frac{1}{\text{Sin. } p} = \text{Cosec. } p.$$

$$\text{Sin.}^2\,\frac{\text{H}}{2} = \text{Cos. } \tfrac{1}{2}\,(a + p + l)\,.\,\text{Sin.}\left\{\tfrac{1}{2}\,(p + l + a) - a\right\}.\text{Sec. } l\,.\,\text{cosec.}\,p.$$

That is

$$\text{Log. Hor. Angle} - 10 = \text{Log. Sec. Latitude} - 10 + \text{Log. Cosec. Polar distance} - 10.$$
$$+ \text{ Log. Cos. } \tfrac{1}{2}\,(a + p + l) - 10 + \text{Log. Sin.}\left\{\tfrac{1}{2}\,(p + l + a) - a\right\} - 10.$$

The formula generally used by navigators using Inman's tables is derived from formula (s), thus:

$$\text{Hav.H} = \sqrt{\text{Hav.}(\text{Z} + (90 - l) - (90 \pm d)}\,.\,\sqrt{\text{Hav.}(\text{Z} - (90 - l) + (90 \pm d)}.$$
$$\text{Cosec. }(90 - l)\,.\,\text{Cosec. }(90 \pm d).$$

Now,

$$\left\{\text{Z} + (90 - l) - (90 \pm d)\right\} = \text{Z} + 90 - l - 90 \pm d = \text{Z} - l \pm d$$

And

$$\left\{\text{Z} - (90 - l) + (90 \pm d)\right\} = \text{Z} - 90 + l + 90 \pm d = \text{Z} + l \pm d$$

Hence

$$\text{Hav. H} = \text{Sec. } d\,.\,\text{Sec. } l\,.\,\sqrt{\text{Hav. }(\text{Z} + (d \pm l))}.\ \sqrt{\text{Hav. Z} - (d \pm l)}.$$

$$\therefore \text{Log. Hav. H} - 10 = \text{Log. Sec. } d - 10 + \text{Log. Sec. } l - 10 + \tfrac{1}{2}\,\text{Log. Hav. }(\text{Z} + (d \pm l)) + \tfrac{1}{2}\,\text{Log. Hav. }(\text{Z} - (d \pm l)) - 10.$$

All the formulæ used in spherical trigonometry are equally capable of proof, and to anyone with a taste for mathematics these investigations are most interesting. As we have seen in these proofs the formulæ have been varied in works on navigation with an idea no doubt of simplifying the calculations; but I cannot help thinking that it would have been better to have left them as they were, and allowed problems in nautical astronomy to be solved by employing the sides of the spherical triangles instead of their complements, and thus sacrificing uniformity and facility of comprehension to utility and rule of thumb.

CHAPTER IX.

MAGNETISM AND THE MARINER'S COMPASS.

MAGNETISM is a subtle natural agency peculiar to certain bodies containing iron. Its existence was first observed as the property of a mineral substance composed for the most part of iron and oxygen, under the form of two oxides, the black and the grey, and commonly called the loadstone. This remarkable compound possesses the power of attracting small particles of iron or steel, and it can also communicate this power to other bodies of a similar nature. The attractive force is found to exist principally at opposite points of the loadstone. These points are termed its poles, from another extraordinary property it possesses —namely, that if a loadstone or natural magnet be suspended by a piece of string or silk, it will always assume a definite position with regard to the terrestrial meridian, either coinciding with it or diverging slightly from it, but in any particular place always assuming the same relative position. This quality is known as the polarity of the magnet. The point of the magnet which is attracted to the north is called its north or red pole, and the opposite point its south or blue pole.

In the construction of the mariner's compass steel magnets are usually employed; they are bars of steel to which magnetism is imparted. A remarkable difference exists between soft or wrought iron, and hard, that is, cast iron, or steel. Soft iron is readily magnetised, and as readily parts with its magnetism. Cast iron or steel requires long-continued application of the magnet to obtain its magnetism, but, once having acquired it, it becomes a permanent magnet.

Magnetism may be imparted by any one of the following methods:

1. Place the bar to be magnetised in a horizontal position. Take two magnets, and place the red pole of one and the blue pole of the other near the centre of the bar, draw them along its surface in opposite directions to each end of the bar, and then slide them back to the middle again without allowing them to leave its surface. After this operation has been repeated often enough the bar becomes a magnet, its red pole being that end which has been rubbed by the blue pole of one of the other magnets, and *vice versá*.

2. In England, or in any place in high northern or southern latitudes, a bar of iron or steel held upright and struck a heavy blow on its end with a wooden mallet will acquire magnetism, its top end having blue and its bottom end red polarity in north latitudes, and the reverse of this in south latitudes.

3. Magnetism may also be induced in such a bar by passing a current of electricity through a wire surrounding it.

It is found that the red pole of one magnet repels the red pole of another magnet, but attracts its blue pole. Hence the fundamental law that *any pole of a magnet will* *repel a like pole, but will attract an unlike pole of another magnet.* The red pole of a magnet is attracted to the north because the earth herself is a huge magnet, whose blue pole is at her north magnetic pole. This pole, in the direction of which the compass needle always points, does not coincide with the true north pole of the earth, but is situated in about latitude 78° N. and longitude 60° W., while the south magnetic pole is in about latitude 76° S. and longitude 130° E. The angular difference between a terrestrial meridian and the direction of the compass needle when free from local disturbance is called *the variation of the compass,* and is named westerly if the needle is drawn to the left of the true north, and easterly

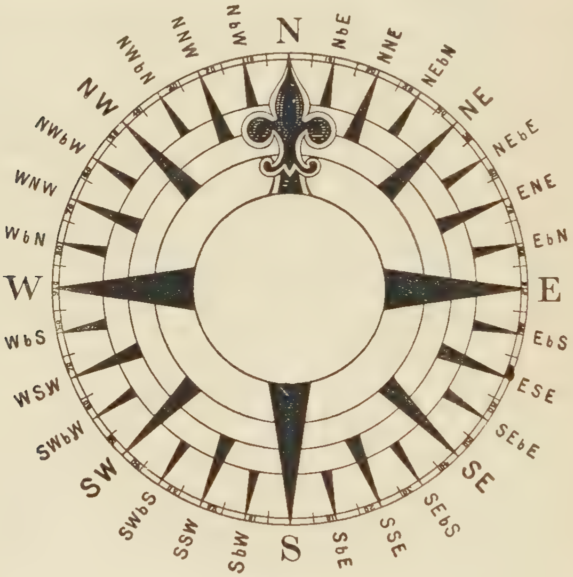
N
NbE
NNE
NEbN
NE
NEbE
ENE
EbN
E
EbS
ESE
SEbE
SE
SEbS
SSE
SbE
S
SbW
SSW
SWbS
SW
SWbW
WSW
WbS
W
WbN
WNW
NWbW
NW
NWbN
NNW
NbW

FIG. 15.

if drawn to the right. The angle between the magnetic meridian and the direction of the compass needle is called *the deviation of the compass*, named westerly or easterly, as the north end of the needle is drawn to the left or the right of the magnetic meridian. Charts of the world are published in which lines of equal variation are drawn. By the use of such a chart it is easy to see what is the error of the compass owing to this cause in any part of the world.

If we balance a knitting needle horizontally by a piece of silk fastened to its centre, and then magnetise it, we shall find that the needle will no longer remain horizontal, but will take a position in this latitude inclined to the horizon at an angle of about 67°. This is called the dip of the needle, and shows the direction in which the magnetic force acts in this country, and in others situated at an equal distance from the magnetic pole.

The dipping needle, as it is called, nowhere assumes a horizontal position except upon a line irregularly drawn round the world, about equidistant from the magnetic poles, and called the magnetic equator. In south latitudes the blue pole dips. At the magnetic poles themselves the dipping needle assumes a perpendicular position, and the magnets in the mariner's compass become useless as directive agents.

The mariner's compass consists of one or more magnets, fastened to a circular card, called the compass card, which is marked round its rim in points and degrees, the line joining the north and south points on the card being parallel to the magnets; it is pivoted on its centre, and made to remain in a horizontal position by weighting it, and by certain mechanical arrangements.

In Fig. 15 we have a representation of the compass card. The cardinal points are north, south, east, and west; the quadrantal points are N.E., S.E., S.W., and N.W. The circumference is marked in degrees, the north and south points being each marked 0, and the east and west points each 90°, the quadrantal points being marked each 45°, and the intermediate spaces marked accordingly. By dividing 360° by 32 we find that each point of the compass is equal to 11° 15′ 0″.

The earth being a magnet, it induces magnetism in all things on its surface. When an iron ship is being built, the hammering which she undergoes causes magnetism of a more or less permanent character to be induced in her. This is known as sub-permanent magnetism, because, though a ship rarely loses it altogether, it alters very much after the vessel is launched, through change of position, through being knocked about in a heavy sea, and from other causes.

In the case of a ship built head south in northern latitudes her blue polarity will be in her bow, and the north point of her compass needle will be attracted to it. This will cause westerly deviations as the ship's head passes through the western half of the compass, and easterly when through the eastern.

If her head is north when building her stern will have blue polarity, and she will have easterly deviation with her head in the western semicircle of the compass, and westerly deviation with her head in the eastern semicircle.

With her head east when building she will have more blue polarity in her starboard side than in her port, and consequently the red end of the needle will be attracted to the starboard, causing easterly deviation with the ship's head in the northern semicircle, and westerly deviation when it is in the southern semicircle. And with her head west when building we shall have easterly deviation on southerly courses and westerly deviation on northerly.

These are called semicircular deviations because they cause westerly deviation in one semicircle of the compass, and easterly in the other; and as they are frequently large it is necessary to adopt means to correct the needle by introducing an equal and opposite force to counteract the effect produced by the sub-permanent magnetism of the ship. The following is the practical method generally adopted: Bring the ship's head to east correct magnetic. If there is westerly deviation, that is if the compass needle is drawn to the left by the sub-permanent magnetism of the ship, place a magnet fore and aft on the port side of the deck, with its red pole aft and its centre on a line with the centre of the compass and at some distance from it. Then move it gradually nearer to the compass till its north point is correct magnetic, fasten the magnet in this position. The correction is made by the red pole of the magnet repelling and the blue pole attracting the red pole of the compass needle. Next, bring the ship's head to north correct magnetic, and correct the compass in a similar manner with a magnet placed on a thwartship line. The effect of the ship's magnetism on the compass is thus counteracted sufficiently to be manageable.

But there is another cause of error, and that is in the temporary magnetism induced in the soft iron of the ship. This is called quadrantal deviation, because it produces similar deviation when the ship's head is in opposite quadrants of the compass card. Thus, if the deviation due to this cause is westerly, or the red end of the needle is drawn to the left when her course lies between north and east, it will be also westerly when she is heading from south to west. The deviation in the other quadrants, namely, those from north to west and from south to east, will be easterly. Soft iron fore and aft, athwartships, and at any angle in a horizontal plane, but either above or below the plane of the compass, causes this error, which is corrected by placing soft iron on either side of the compass box so that the plane of the compass will pass through the centre of the soft iron used.

Vertical iron should never be permitted near the compasses. In high latitudes it becomes, by induction, a powerful magnet, and has a very great effect on the compass needle. If it is unavoidable, compensation must be made by placing, on the opposite side to this vertical iron, a vertical bar of soft iron. When this has been done accurately, the correction will be good for any position of the ship on the earth's surface, because the same magnetism will be induced in the vertical iron and in the corrector, and so they will counteract each other.

A constant deviation has sometimes to be allowed on all courses. It is due either to want of accuracy in the lubber line, or to the joint action of unsymmetrical iron, and is generally extremely small.

In northern latitudes, a heel to port usually causes easterly deviation in the northern semicircle, and westerly in the southern. A heel to starboard produces a contrary effect. Heeling error is due to sub-permanent and to induced magnetism. As horizontal soft iron is more or less inclined, it becomes more or less magnetised by induction. The position of the ship generally with regard to the plane of the compasses will cause an alteration in its effect upon them. It will thus be seen that, as we approach the equator, the soft iron beams, &c., will become less and less affected by the induced magnetism caused by the heel of the ship, and that in southern latitudes the magnetism will be of an opposite nature to that produced in the northern. A compass in north latitudes is compensated for heeling error by placing a vertical magnet with its red pole up under the centre of such compass. Heel the ship and raise or lower the magnet till the error caused is rectified. It must be observed that, generally, as the ship approaches the equator, heeling error, due to induced magnetism, gradually disappears, till it becomes *nil* at the equator, and assumes an opposite character as she gets into southern latitudes. The compensating magnet must then be turned round, with its blue pole up.

The true course of a vessel is the angle which her track makes with a meridian of the earth. We must note here that a line drawn on the earth's surface, cutting each meridian at the same angle, is a curve, and is called a rhumb-line.

The correct magnetic course is the angle between the direction of a ship's head and the magnetic meridian, or line passing through the place of the ship and the magnetic pole.

The compass course is the direction of the ship's head by the standard compass.

Leeway is the angular distance between the direction of a ship's head and the track she actually makes owing to the pressure of wind acting on either beam. This correction is always allowed to the left if the wind is on the starboard beam, and to the right if on the port beam.

In correcting courses it is necessary to imagine oneself standing in the centre of the compass card, and looking thence in the direction of the course to be corrected. From that position east is to the right of north and west is to its left; also east is to the left of south and west to its right. It is most important to get a clear idea of this, and it would be well for my readers to get a plate of the compass card and see for themselves that, for instance, N.E. is right of north, E.N.E. is right of N.E. but left of east; that S.S.E. is left of south and right of S.E.; that W. by N. is right of west; that S.S.W. is right of south and left of S.W., &c.

In converting compass courses into true courses variation and deviation are allowed to the left when westerly, to the right when easterly. For example: A ship has been steering east by compass, the variation being $1\frac{3}{4}$ points W. and deviation $\frac{1}{4}$ point E., the wind N.N.E. and leeway $\frac{1}{2}$ point. What is the true course?

Compass course	East = 8 points right of north
Variation	$1\frac{3}{4}$ W. left.
Magnetic course	E. by N. $\frac{3}{4}$ N. $6\frac{1}{4}$ points right of north.
Deviation	$\frac{1}{4}$ E. right.
True course if no leeway	E. by N. $\frac{1}{2}$ N. = $6\frac{1}{2}$ points right of north
Leeway	$\frac{1}{2}$ right.
True course	E. by N. = 7 points right of north.

Observe that we have $1\frac{3}{4}$ points left variation, $\frac{1}{4}$ point right deviation, and owing to the direction of the wind,

$\frac{1}{2}$ point right leeway ; or $1\frac{3}{4}$ left $-\frac{1}{4}$ right $-\frac{1}{2}$ right $=1$ point left. It saves trouble to take the sum or difference between the variation and deviation as they have like or unlike names, and, with the sign of the larger amount, apply the correction to the compass course.

Compass course.	Variation.	Deviation.	Sum or difference	True course.
N.E.	$1\frac{1}{2}$ E.	$\frac{1}{2}$ E.	2 E. right	E.N.E.
W.N.W.	$1\frac{3}{4}$ E.	$\frac{1}{2}$ W.	$1\frac{1}{4}$ E. right	N.W. $\frac{3}{4}$ W.
S.E. by E.	$1\frac{1}{2}$ W.	$\frac{3}{4}$ W.	$2\frac{1}{4}$ W. left	E. $\frac{3}{4}$ S.
S. by W.	$2\frac{1}{2}$ W.	$\frac{1}{4}$ E.	$2\frac{1}{4}$ W. left	S. by E. $\frac{1}{4}$ E.
W. by S.	$\frac{1}{2}$ E.	$\frac{3}{4}$ W.	$\frac{1}{4}$ W. left	W. by S. $\frac{1}{4}$ S.

If any leeway had been logged for any of these courses, it would have to be applied with its right sign to the true course here found.

Suppose, now, we know the true course, and we wish to find the corresponding compass course to be steered, the above corrections must be applied the reverse way. Thus, easterly variation and deviation must be applied to the left, and westerly to the right.

EXAMPLES :

True course.	Variation.	Deviation.	Sum or difference.	Compass course.
N.E. by N.	$1\frac{3}{4}$ E.	$\frac{1}{2}$ W.	$1\frac{1}{4}$ E. left	N. by E. $\frac{3}{4}$ E.
S.E. $\frac{1}{2}$ E.	$1\frac{1}{2}$ W.	$\frac{1}{2}$ W.	2 W. right	S.S.E $\frac{1}{2}$ E.
S. $\frac{1}{2}$ W.	2 W.	$\frac{1}{4}$ E.	$1\frac{3}{4}$ W. right	S.S.W. $\frac{1}{4}$ W.
W. by N.	$2\frac{1}{2}$ E.	$\frac{3}{4}$ W.	$1\frac{3}{4}$ E. left	W. $\frac{3}{4}$ S.

After a ship's compasses have been compensated in the manner described above, the residual error or deviation requires to be found and tabulated for each position of the ship's head. The deviation may be found by (1) reciprocal bearings, (2) shore marks in a line with a conspicuous object, (3) equal bearings of a distant object, (4) by the line of bearing of two objects whose positions are known, and (5) by azimuths. The first two methods are employed when a ship is in harbour. The last three when she is at sea.

1. Having made arrangements to swing the ship round to all points of the compass, an azimuth compass is stationed on shore at some point well clear of any disturbing magnetic influence, so that the bearings taken by it will be correct magnetic. Now we bring the ship's head north by standard compass, and take a bearing of the compass on shore, an observer at that compass taking a bearing of the standard compass on board at the same moment. Her head is then brought to N. by E., and the operation repeated, and so on for every point. We will suppose that the bearing of the shore compass by the standard when the ship's head was north to be N.E. ½ N. At the same time, the bearing of the standard compass from the shore was S.W. Before comparing these bearings, it is necessary to change one into the opposite bearing. We will, therefore, change the bearing by the shore observer from S.W. to N.E. Now, N.E. is the correct magnetic bearing, and N.E. ½ N. is the compass bearing from the ship's standard compass. The difference (half point) is the deviation, and it is easterly because the correct magnetic bearing is to the right of the compass bearing. A deviation table is then constructed thus :

Ship's head by standard compass.	Deviation.
North	½ point E.
N. by E.	¾ point E.
N.N.E.	¾ point E.

and so on for every point. *It must be clearly understood that the deviations are made out for the direction of the* *ship's head by compass.* The deviation in the table is always given in degrees. In practice the course steered is usually in points and quarter points, as, owing to the motion of the ship, it is practically impossible, except in perfectly smooth water, to make a more accurate course. But the course is generally corrected to degrees, as we shall see hereafter.

2. At Liverpool and, I believe, in some other ports, the dock wall is marked in degrees, &c. These, when brought on with a large chimney at some distance behind the dock wall, give its correct magnetic bearing, and thus the deviation for the different positions can be ascertained.

3. At sea the bearings of a conspicuous object at a distance of not less than ten miles by the standard compass, taken when the ship's head is north, N.E., east, S.E., south, S.W., west, and N.W., will enable us by the use of a Napier's curve to form a deviation table. The mean of the eight bearings will give the correct magnetic bearing. This is found by taking the sum of these bearings and dividing them by eight if in the same quadrant. If part are in one quadrant and part in another, but all in the northern or southern half of the compass, the difference must be taken between their respective sums and divided by eight, the bearing found being in the same quadrant as the larger sum :

Ship's head by standard compass.	Bearing of distant object.	Ship's head by standard compass.	Bearing of distant object.
North	S. 16° E.	South	S. 14° W.
N.E.	S. 1° W.	S.W.	S. 11° E.
East	S. 15° W.	West	S. 26° E.
S.E.	S. 20° W.	N.W.	S. 26° E.

Here the bearings are all in the S.E. and S.W. quadrants.

In the S.E. quadrant.	In the S.W. quadrant.
16°	1°
11°	15°
26°	20°
26°	14°
79°	50°
50°	
8)29°	
3⅝°	

Correct magnetic bearing S. 3½° E. very nearly.

because the sum of the south-easterly bearings is the greater.

Ship's head by standard compass.	Compass bearing.	Correct magnetic bearing.	Deviation.
North.........	S. 16° E.........	S. 3½° E.	12½° E.
N.E.	S. 1° W.	S. 3½° E.	4½° W.
East	S. 15° W.	S. 3½° E.	18½° W.
S.E.	S. 20° W.	S. 3½° E.	23½° W.
South	S. 14° W.	S. 3½° E.	17½° W.
S.W.	S. 11° E.........	S. 3½° E.	7½° E.
West	S. 26° E.........	S. 3½° E.	22½° E.
N.W.	S. 26° E.........	S. 3½° E.	22½° E.

With the deviation of the compass thus ascertained, the curve on the Napier's diagram at the end of this work is drawn.

If the bearings are in different quadrants, but all in the eastern or western half of the compass, they must all be turned into bearings either from the north or south point of the compass, and their mean found by adding them together and dividing the sum by 8. Thus, suppose the bearings to be S. 79° E., S. 84° E., east, N. 85° E., N. 80° E., N. 77° E., N. 84° E., and N. 89° E. As there are fewer bearings in the S.E. quadrant they must be turned into N.E. bearings by subtracting each of them from 180°.

Then S. 79° E. = N. 101° E.
S. 84° E. = N. 96° E.
and East = N. 90° E.

We have, therefore, to add together

101°
96°
90°
85°
80°
77°
84°
89°
and divide by 8)702°

The mean bearing is N. $87\frac{3}{4}$° E.

THE CONSTRUCTION OF A NAPIER'S DIAGRAM.

The central perpendicular line is the rim of a compass card drawn in a straight line, on which the points of the compass and the degrees are marked. These points are named from up, down, north, N. by E., and so on, round by east to south, then west, and so back to north. Through each point two lines cross each other, making an angle of 60° with the vertical line and with each other. One of these lines is dotted, and has its upper end on the left of the diagram. The other, a plain line, has its upper end on the right side. All these lines form with the vertical line a series of equilateral triangles. If, with a pair of dividers, we measure any distance from the perpendicular line along a plain line, the same distance will take us back to the perpendicular line along or parallel to a dotted line, and *vice versâ*. We will first show how the deviations from the several points in the example given above are laid off; and then how the deviation curve is described, and the use which we can make of it.

With the ship's head north, the deviation is 12½° E. Measure these degrees off with a pair of dividers on the central line, then from the north point lay this distance off along the *dotted line* to the *right*, because the deviation is east, and mark the spot. From the N.E. point mark off 4½° W. along the *dotted line* to the *left*, because the deviation is in this case west. And in the same manner set off the deviations from their respective points, making marks as described above. Then join all these marks by a freely drawn curved line. This line is the deviation curve.

To find the correct magnetic course from a compass course, measure along the dotted line, or a line parallel to it, from the compass course given to the deviation curve, and return to the perpendicular line on a plain line or on a line parallel to it. The point which you thus reach will be the correct magnetic course required. Thus compass course E.N.E., to find magnetic course. It is N.E. b. E. a little northerly or N. 56° E. Again, compass course east is N. 72° E. magnetic.

To find compass course from magnetic, the rule is to measure to the curve along the plain line and return by the dotted line. Thus correct magnetic course west, the compass course to be steered will be S. 71° W.; or correct magnetic course S. b. E., the compass course to be steered will be S. 4° W.

To correct compass course go along dotted, return by plain.

To find compass course go along plain, return by dotted, are the golden rules for using a Napier's diagram and deviation curve.

The deviation on any particular position of the ship's head may be measured on a Napier's curve from the centre line along or parallel to a dotted line to the curve.

4. When the correct magnetic bearing of two objects in a line is known, it may be used for ascertaining the deviation. Such objects are the Lizard lighthouses, Portland lighthouses, &c.

5. Lastly, the deviation may be ascertained by azimuth of the sun or other heavenly bodies.

DEVIATION TABLE.

This table is founded on the Napier's curve, used in this work, except where special deviations are given :

Ship's head by standard compass.	Deviation.	Ship's head by standard compass.	Deviation.
North	12½° E.	South	17½° W.
N. by E.	9° ,,	S. by W.	12½° ,,
N.N.E.	5° ,,	S.S.W.	7° ,,
N.E. by N.	1° ,,	S.W. by S.	*Nil.*
N.E.	4½° W.	S.W.	7½° E.
N.E. by E.	8° ,,	S.W. by W.	14° ,,
E.N.E.	12° ,,	W.S.W.	18° ,,
E. by N.	15° ,,	W. by S.	20° ,,
East	18½° ,,	West	22½° ,,
E. by S.	20° ,,	W. by N.	24° ,,
E.S.E.	22° ,,	W.N.W.	25° ,,
S.E. by E.	23° ,,	N.W. by W.	24° ,,
S.E.	23½° ,,	N.W.	22½° ,,
S.E. by S.	23° ,,	N.W. by N.	21½° ,,
S.S.E.	22° ,,	N.N.W.	19° ,,
S. by E.	20° ,,	N. by W.	16° ,,

It is convenient when sailing in shoal waters to construct a straight course table, so as to be able to turn any magnetic course one wishes to steer into the corresponding compass course by inspection. See Table II. at the end of the work.

CHAPTER X.

Log and Lead.

The log is used for the purpose of ascertaining at what rate per hour a ship is travelling through the water.

In old times it used to be the custom to heave a lump or log of wood from the bows of a vessel and walk aft with it, then very roughly estimating, by a comparison between the rate of walking and the rapidity with which the log floated aft, the velocity of the ship. Hence the name "Log."

The ordinary log consists of a line called the log-line, and a triangular piece of wood, rounded at the bottom and weighted with lead so that it may float in a perpendicular position, which is called the log-ship.

The log-line is permanently attached to the log-ship at two of its corners, and is secured to the other corner by means of a leather or bone plug inserted into a hole, which will come out when the strain becomes excessive, so that the log-ship, after use, may be hauled on board with greater ease.

Let A B C be the log-ship, and L the end of the log-line, which is attached to A and B. Let P be the leather or bone plug which, when the log is about to be hove, must be inserted in the hole at C. The log-ship is weighted with lead along the rim B C, which will cause it to float perpendicularly to the pull of the log-line until the plug is jerked out of the hole, when it can be hauled on board with the least resistance.

When a ship is moving ahead there is always a certain amount of water immediately astern which follows her, called the dead water, and as the principle of the log is that the log-ship should be stationary in the water, a sufficient quantity of line must be allowed to run out to enable the log-ship to get clear of the dead water; this portion of the log-line is called the stray line, and its length depends upon the size of the ship and her height above the water. As a rule, 10 fathoms at least should be

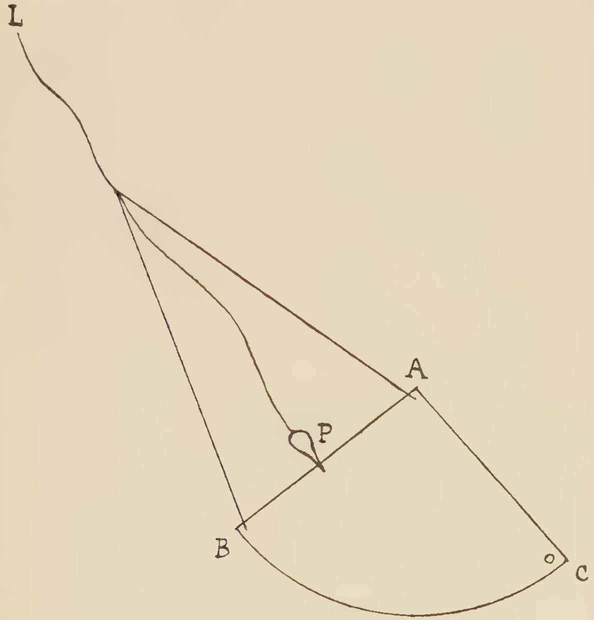

FIG. 16.

allowed from the log-ship; here a piece of white bunting or rag is secured. From this the knots are measured, the length of each one having the same proportion to the length of a nautical mile as the number of seconds the log-glass used takes to run out bears to the number of seconds in an hour. These log-glasses are constructed in exactly the same way as the ordinary egg-boilers or the old-fashioned hour-glasses, and are usually of two lengths, viz., 28 seconds and 14 seconds.

To find the length of the knot for the 28-second glass, we have this proportion:—The number of feet in one knot : the number of feet in one nautical mile :: length of glass in seconds : the number of seconds in an hour. There are 6080 feet in a nautical mile, and 60 × 60 = 3600 seconds in an hour, therefore:

3600 : 28 :: 6080 : length of knot for 28-second glass
28
48640
12160
3600)170240(47ft. 3·47in. nearly
14400
26240
25200
1040
12
12480
10800
16800
14400
24000
25200

which gives us the length of the knot when using the 28-second glass as 47ft. 3·47in. very nearly. At this distance from the white mark on the log-line place a piece of cord with one knot tied in it; at an equal distance from this mark a piece of cord with two knots; and so on up to, usually, 7½ knots. Half-way between each of these knots place a single knot for the half knot.

We have therefore in an ordinary log-line the white mark at 10 fathoms or more from the log-ship, a mark with one knot at the half knot, the same at one knot and at one and a half knots, a mark of two knots at two knots, one knot at two and a half knots, three knots at three knots, and so on.

When using the 14-second glass, of course the distance given by the log-line must be doubled; that is to say, if the five and a half knot mark is in the hand when the 14-second glass is run out, the ship is travelling at the rate of eleven knots per hour. The log-line is wound round a reel, so that it may run out freely.

When it is desired to ascertain the speed of the ship, the reel is held by one person and the log-glass by another, who takes care that one end of it is free of sand. The seaman heaving the log then pushes the leather or bone plug into the hole in the log-ship, and coils in his hand enough line off the reel to allow the log-ship to be thrown well clear of the ship's wake; he then, standing at the lee quarter, throws the log-ship into the sea, well to leeward, having first assured himself that the log-glass is clear, the log-ship will now take the line off the reel. As the white mark passes through his hands he will call out "Turn" to the man holding the log-glass, who at once turns it, and watches it carefully till the sand has all run out, when he cries "Stop," and the log-line is seized by the heaver, and the number of knots and tenths of knots run out indicates the speed of the ship.

The ordinary log, of course, only indicates the rate of sailing when it is being hove, and a ship may have only gone seven miles through the water during an hour, though the log shows that she is going at the rate of ten knots at the end of it. To obviate this cause of inaccuracy many patent logs have been invented. In most cases they consist of a freely revolving screw, drawn through the water by the ship, it being known by calculation how many revolutions it makes in a mile, the number of knots it has been dragged through the water is registered on dials in much the same way as the number of cubic feet consumed in a gas meter.

LEAD.

There are two descriptions of leads used on board ship to ascertain the depth of water, named respectively hand leads and deep sea leads. Hand leads weigh from 7lb. to 14lb., and are used for sounding in shallow water. Deep sea leads weigh 28lb. and upwards, and are employed, as their name indicates, to find the bottom in deep water. They have a cavity in the lower extremity which is filled with hard grease, called the arming of the lead, to which when the lead strikes the bottom a portion of sand, shells, or other substance will adhere, enabling the navigator to compare it with that given on the chart, where small letters are placed, such as *g*, gravel; *m*, mud; *s*, sand, &c., to denote the nature of the bottom of the sea.

The hand lead is 25 fathoms in length, and is marked as follows:

At	2	fathoms	A piece of	leather with two ends.
,,	3	,,	,,	,, three ,,
,,	5	,,	,,	white calico.
,,	7	,,	,,	red bunting.
,,	10	,,	,,	leather with a hole in it.
,,	13	,,	,,	blue serge.
,,	15	,,	,,	white calico.
,,	17	,,	,,	red bunting.
,,	20	,,	A strand	with two knots.

The difference in the material used for the marks is to enable the leadsman at night to know by the feel what sounding he has.

All the fathoms at which there are no marks are called deeps. The hand lead therefore consists of nine marks and eleven deeps. The deeps are at 1 fathom, and at 4, 6, 8, 9, 11, 12, 14, 16, 18, and 19 fathoms.

The deep sea lead line is marked in the same way as the hand lead line as far as 20 fathoms, after which at every ten fathoms a strand is placed, having three knots in it at 30 fathoms, four at 40 fathoms, five at 50 fathoms, and so on; midway between each of these is placed a strand with one knot, marking the 25, 35, 45 fathoms, &c., according to its position.

At 100 fathoms is placed a piece of bunting, and the marking is repeated.

The hand lead is hove by men standing in the chains, who after each cast call out the sounding obtained. Suppose the depth of water is 7 fathoms, the red bunting mark will be just level with the water, and the leadsman will call: "By the mark 7." If there are 12 fathoms of water, the 13-fathom mark (blue serge) will be six feet above the surface of the water, and the leadsman's cry will be "Deep 12." Every fathom not marked is termed a "deep." The lead line must be vertical and taut when the sounding is taken, or the depth of water will be inaccurately measured, and in excess of the truth. Halves and quarters of fathoms are called in the following manner: $5\frac{1}{4}$ fathoms, "And a quarter five"; $5\frac{1}{2}$ fathoms, "And a half five"; $5\frac{3}{4}$ fathoms, "A quarter less six"; &c.

To heave the common deep sea lead, the ship is usually hove to. The reel round which the lead line is wound is taken up to the lee quarter of the vessel, the end of the line is passed forward outside and clear of everything to the lee cathead, where it is secured to the deep sea lead, which has been well armed with grease. The seaman holding it coils in his hand sufficient line to enable him to cast the lead well from him to leeward. A line of seamen is formed along the outside of the ship to leeward, each of whom holds a coil of the lead line in his hands. If the sounding is known to be less than 90 fathoms, 100 fathoms should be reeled off. When all is ready, the order "Heave" is given, and the man at the cathead heaves the lead as far as he can away from the ship, calling out: "Watch, there watch." When the line is nearly run out from the next man he repeats the cry, which is passed from man to man till the lead strikes the bottom, whereby the sounding is obtained. The lead is then hauled on board, care being taken not to allow it to strike against the ship's side, in case the arming should be displaced. Then the nature of the bottom, ascertained by the matter attached to the grease, is noted and compared with the chart.

This cumbrous and somewhat inaccurate method has been almost entirely superseded by various patented arrangements, which enable the mariner to get the same results without stopping the ship. The best and most modern are those which depend for their accuracy upon the pressure of the water at different depths. A glass tube open at one end descends with the lead, the air contained in it is compressed by the water, or, in other words, the water ascends into the tube to a greater or less height in proportion to the depth. On reaching the bottom the water is retained in the tube by means of a valve, and on being hauled on board its height in the tube indicates by a scale the depth of water attained. To accelerate the descent of the lead, the lead line, which has some buoyancy, is replaced in the best instruments by piano wire. This is coiled round a drum, and when sufficient wire has been run out to ensure the lead touching the bottom, the drum is stopped by a break, and the wire wound up by means of handles to haul the lead on board.

CHAPTER XI.

DEAD RECKONING.

BY dead reckoning is meant the method of navigating a vessel without astronomical observations. In Fig. 17 let

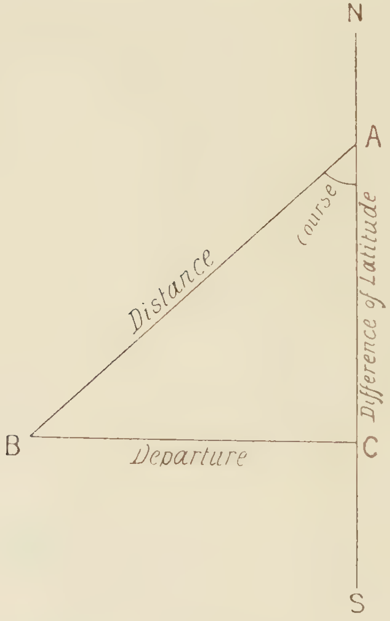
N
A
Course
Distance
Difference of Latitude
B
Departure
C
S

FIG. 17.

N S be a meridian. Suppose a ship to sail from A to B; the difference of latitude is A C, the departure C B, the distance the ship has sailed A B, and the course the angle B A C. Now, the triangle A B C being a right angled triangle, if we are given any two parts we can find the rest. See chapter on trigonometry. We know the distance A B and the course angle A; therefore we can find the difference of latitude and the departure. Let the latitude of A be 49° 16′ N., and its longitude 14° 29′ W., the course sailed S. 54° W., and the distance 129 miles.

	A C = A B × Cos. A.		C B = A B × Sin. A.
Diff. lat......	= Dist. × Cos. Course.	Departure	= Dist. × Sin. Course.
	= 129 × Cos. 54°.		= 129 × Sin. 54°.
Log. 129.....	= 2·110590	Log. 129	= 2·110590
Log. Cos. 54°	= 9·769219	Log. Sin. 54°	= 9·907958
Log. Diff. lat.	= 1·879809	Log. Departure	= 2·018548
Diff. lat.	= 75·82 miles nearly.	Departure	= 104·4 miles nearly.

To Find the Latitude Arrived At.

In sailing from A to B the diff. lat. is 75′·82, practically 76′, which is equal to 1° 16′ to the southward. This is applied to the latitude of A as follows: If the ship has run to the southward in north latitude, as in this example, it is to be subtracted from the latitude of A; but if to the northward it must be added. The reverse of this occurs in southern latitudes. Therefore we have

Latitude of A......... = 49° 16′ N
Diff. Lat............... = 1° 16′ S
Latitude of B......... = 48° 0′ N

To Find the Difference of Longitude from the Departure.

We have seen that the departure is the actual easting or westing made. In this case it is 104·4 nautical miles nearly. The problem is how to obtain the difference of longitude due to this departure, in order to find the longitude of B. It is obvious that the further we are from the equator the greater will be the change in longitude for a given departure, because the meridians converge to the poles where they meet; and sixty miles of departure will be more than 1° of longitude everywhere, except on the equator. The formula employed is Diff. long = Departure × Secant latitude, or Departure = Diff. long. × Cosine latitude, and it is proved thus:

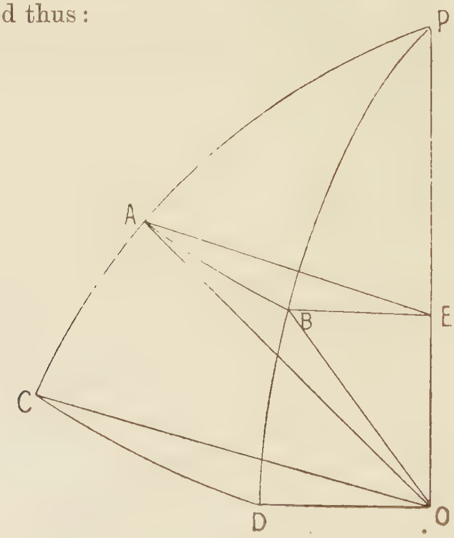

Fig. 18.

Let Fig. 18 represent a segment of the earth, P being the pole, O the centre, C D part of the equator, A B part of a parallel of latitude, P D and P C portions of two meridians passing through A and B, and E the centre of the parallel of latitude A B. A B is the departure between A and B, and C D is their difference of longitude. We have seen that arcs of circles are directly proportional to their radii therefore A B : C D :: B E or A E : D O or C O. Now D O, C O, B O, and A O are all equal one to the other, because they are all radii of the earth.

$$\therefore AB : CD :: BE : OB \text{ or } \frac{CD}{AB} = \frac{OB}{BE}$$

but (B E O being by construction a right-angled triangle) $\frac{OB}{BE}$ = cosec. B O E which equals sec. B O D, because B O D is the complement of B O E. And we have

$$\text{Sec. } BOD = \frac{OB}{BE} \text{ but } \frac{OB}{BE} = \frac{CD}{AB}$$

$$\text{Sec. } BOD = \frac{OB}{BE} = \frac{CD}{AB} = \frac{\text{Diff. long.}}{\text{Departure.}}$$

And the angle B O D = the arc B D, the latitude.

$$\therefore \text{Secant latitude} = \frac{\text{Diff. long.}}{\text{Departure.}}$$

$$\text{Or Cosine latitude} = \frac{\text{Departure.}}{\text{Diff. long.}}$$

Mid Latitude Sailing.

The departure in the above problem 104·4 miles has been made in going from Lat. 49° 16′ N. to Lat. 48° 0′ N. It is obvious that if we use Lat. 49° 16′ to find the diff.

long. it will be too large, and if Lat. 48° 0′ too small, so the middle or mean latitude is employed.

Lat. A... 49° 16′ N.
Lat. B... 48° 0′ N.
2)97° 16′
Mid. lat. 48° 38′

Now diff. long. = Depart × Secant. mid. lat.
= 104·4 × Sec. 48° 36′
Log. 104·4 = 2·018548
Log. sec. 48° 38′ = 10·179880
Long. diff. long. = 2·198428
Diff. long. = 157·9 nearly
= 2° 38′ W.

which we apply to longitude of A thus, and so find longitude of B.

Long. of A = 14° 29′ W.
Diff. long. = 2° 38′ W
Long. of B. = 17° 7′ W

It is customary, however, to find the diff. long. from the departure by inspection in this way. In the example above given the departure is 104·4, and the mid. lat. 48° 36′. In the Traverse Table, Inman, page 21; Norie, page 57, we find that, taking the nearest degree of latitude, 49°, as a course, and the departure, 104·4, in the diff. lat. column, the distance corresponding 158′ is the diff. long. required. Let us try a few more examples :

	Departure.	Mid. lat.	Diff. long. by inspection.
Inman, page 20 Norie, ,, 53	121	37°	152′ = 2° 32′
Inman, ,, 11 Norie, ,, 26	75	10°	76′ = 1° 16′
Inman, ,, 28 Norie, ,, 43	98	63°	216′ = 3° 36′

Suppose we are given the latitude and longitude of A and B as Lat. 10° 20′ S., Long. 120° 15′ E., and Lat. 13° 48′ S., Long. 121° 10′ E. respectively, to find the diff. lat. and departure made in sailing from A to B.

Lat. A ...	10° 20′ S.,	Long. A ...	120° 15′ E.	Lat. A ...	10° 20′ S.,
Lat. B ...	13° 48′ S.,	Long. B ...	121° 10′ E.	Lat. B ...	13° 48′ S.,
Diff. lat. =	3° 28′ S.,	Diff. long..	0° 55′ E.		2)24° 8′
	60			Mid. lat.	12° 4′
„ =	208′ S.				

The diff. lat. is marked south and the diff. long. east, because the ship has gone south and east.

By the formula as shown above, p. 100 :

$$\text{Cos. lat.} = \frac{\text{Departure}}{\text{Diff. long.}}$$

$$\begin{aligned}\therefore \text{Departure} &= \text{Diff. long.} \times \text{Cos. mid. lat.}\\ &= 55' \times \text{Cos. } 12° \, 4'\end{aligned}$$

$$\begin{aligned}\text{Log. } 55' &= 1{\cdot}740363\\ \text{Log. Cos. } 12° \;\, 4' &= \underline{9{\cdot}990279}\\ \text{Log. Departure} &= 1{\cdot}730642\\ \text{Departure} &= \;\; 53'{\cdot}78\end{aligned}$$

We can also find this by inspection by taking in the Traverse Table the diff. long. as the distance and the mid. lat. as the course. The amount in the diff. lat. column is equal to the departure.

Inman, page 8, Norie, page 28, diff. long. 55′; mid. lat. 12°; departure by inspection, 53′·8.

We have now the diff. lat. 208′ S., and departure 53′·78 E., and by the right angled triangle, Fig. 17, we can find the course and distance from A to B.

$$\text{Tangent course} = \frac{\text{Departure}}{\text{Diff. lat.}}$$

$$\text{and Distance} \qquad = \text{Diff. lat.} \times \text{Secant course.}$$

Tan. course = $\frac{53'\cdot78}{208'}$

Log. 53′·78 = 1·730621
,, 208′ = 2·318063

Log. tan. course = 9·412558

Distance = 208′ × Secant course.

Log. 208′ = 2·318063
Log. sec. 14° 29′ 45″ = 10·014050

Log distance = 2·332113

Course A to B = S. 14° 29′ 45″ E., distance = 214′·8 nearly.

From the Traverse Table we can also obtain an approximate course and distance (see page 28 Inman, pages 30 and 31 Norie), with the diff. lat. and departure found above.

The Traverse Table is simply the solutions of right-angled triangles formed by the courses to each point and degree on the compass card, distances to 296′ in Inman and 300′ in Norie, and their respective diff. lats. and departures tabulated, to save navigators the time and trouble of working them out by logs.

Parallel Sailing.

Parallel sailing is finding the diff. long. from the departure, which we have already done. Let us take an example.

A ship in latitude 51° 45′ N. and longitude 13° 55′ W. sailed West true 140′. Find her longitude.

Diff. long. = Departure × Sec. latitude.
= 140′ × Sec. 51° 45′

Log. 140......... = 2·146128
Log. sec. 51° 45′ = 10·208243

Log. diff. long.. = 2·354371
Diff. long ...... = 226·1 = 3° 46′ W.

Long. left....... = 13° 55′ W.
Diff. long ...... = 3° 46′ W.

Long. in ...... = 17° 41′ W.

MERCATOR'S SAILING.

Let $M M_1$, Fig. 19, be part of a meridian. A ship sails from A to B; her course is B A C, distance A B, diff. lat. A C, and departure B C. Now extend A B to E, and make E D perpendicular to $M M_1$, equal to the diff. long. between A and B, then A D will evidently be the meridional

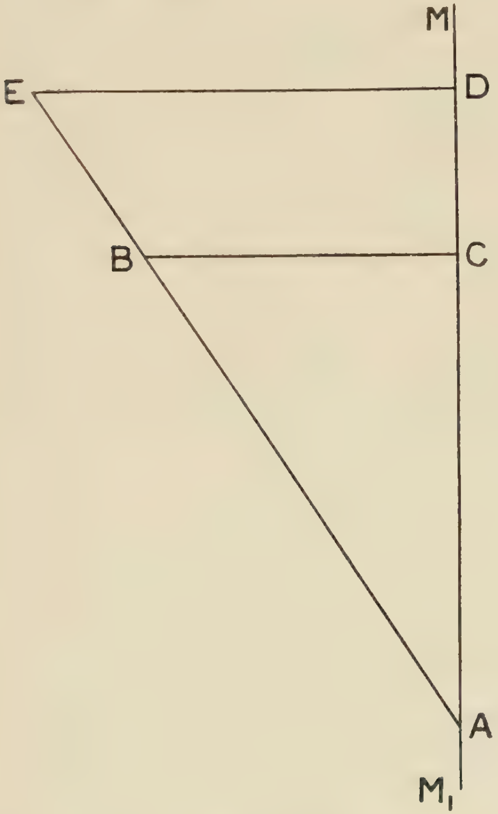

FIG. 19.

difference of latitude, as by the construction of a Mercator's chart the distance between the parallels of latitude is increased in the same proportion as the distance between the meridians in any latitude. (*See* page 51.)

Now suppose we want to find the course and distance from A to B. In the triangle A E D tan. A = $\frac{\text{E D}}{\text{A D}}$, but A or B A C is the course; that is, tan. course = $\frac{\text{Diff. long.}}{\text{Mer. diff. lat.}}$; and in triangle A B C, A B = A C × Sec. A, that is, distance = diff. lat. × sec. course. For example:

Find the course and distance from Bilbao (lat. 43° 22′ N., long. 3° 4′ W.) to Sandy Hook (lat. 40° 28′ N., long. 74° 0′ W.).

Mer. parts :*

Lat. fr. 43° 22′ N.................... 2893·27 ....Long. fr. 3° 4′ W.
Lat. to 40° 28′ N.................... 2659·37 ....Long. to 74° 0′ W.

Diff. lat. 2° 54′ S. Mer. diff. lat. = 233·90 Diff. long. 70° 56′ W.
60 60
174′ S. 4256′ W.

Tan. course = $\frac{\text{Diff. long.}}{\text{Mer. diff. lat.}}$	Distance = Diff. lat. × Sec. course.
∴ Tan. co. = $\frac{4256}{233·9}$	∴ Distance = 174 × Sec. course.
Log. 4256′ = 3·629002	Log. 174′ = 2·240549
Log. 233′·9 = 2·369030	Log. sec. course = 11·260605
Log. tan. co. = 11·259972	Log distance = 3·501154
	059
	95
Course = S. 86° 51′ 15″ W.	Distance... = 3170·7

---

* Inman Table 7, Norie Table 3.

In this example the diff. lat. is marked south because Sandy Hook is in less N. latitude than Bilbao, and the longitude is marked W. because Sandy Hook is in greater W. longitude. If the latitudes of the two places differ in name it is evident that the diff. lat. will be their sum, and the mer. diff. lat. the sum of their meridional parts. If they are of the same name it will be their difference. Again, if the longitude of A is 171° 15′ E, and that of B 163° 10′ W., the diff. long. will be $360° - (171° 15' + 163° 10')$, that is 25° 35′; and if we are sailing from A to B it must be marked E. In Fig. 20 let G Q M Q_1 be the equator and P the north pole. We are supposed to be looking down

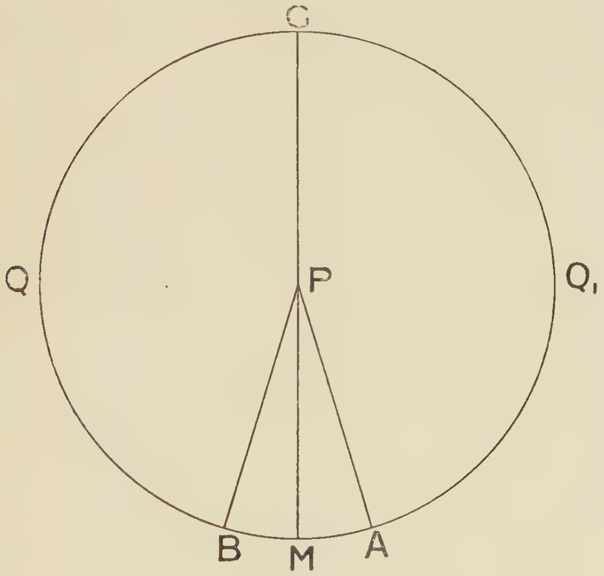

FIG. 20.

on the globe from an immense height at a point situated exactly over the north pole. Let P G be the prime meridian or meridian of Greenwich, P M the meridian 180° from Greenwich, and therefore exactly opposite to it, P B the meridian of B, and P A the meridian of A. Then G Q B is the longitude of B east, and $G\,Q_1\,A$ is the longitude of A west. It is obvious that their diff. long. is B M A; this arc is equal to $B\,M + M\,A = 180° - G\,Q\,B + 180° - G\,Q_1\,A$ (since G Q M and $G\,Q_1\,M$ are each equal to 180°) $= 360° - G\,Q\,B - G\,Q_1A = 360° - (G\,Q\,B + G\,Q_1\,A) \therefore$ diff. long. = 360° − (long. A + long. B), and it is marked in this case W, because a ship in sailing from A to B will go to the westward, though she goes from west longitude to east longitude.

The rule then is that the diff. long. between two places situated on different sides of the prime meridian is the sum of such longitudes except when this exceeds 180°, then it must be subtracted from 360°, as shown above, to find such diff. long. In the first case the diff. long. is marked east or west, according to the longitude of the place arrived at; in the latter it takes the same name as the longitude of the place left.

Thus:	Long. from	Long. to	Diff. long.
	10° W.	10° E.	20° E.
	10° W.	175° E.	175° W.
	140° E.	140° W.	80° E.
	99° W.	170° E.	91° W.

The Day's Work.

On board a ship the officer of the watch causes the log to be hove every hour to ascertain the rate at which she is sailing, and he enters on the log-slate the course she has steered, the distance that he estimates she has travelled, and any leeway she may have made. In a sailing vessel the speed during an hour often varies considerably, particularly when the wind is not steady; but with experience a very accurate register of hourly distances run may be kept.

Overleaf is the usual form of the day's work, which is calculated up at noon on each day. In the first column are the hours. In the second and third, the knots and tenths of a knot which it is estimated that the ship has sailed in the hour. In the fourth column the course the ship has made by standard compass. In the fifth the leeway, according to the officer of the watch's judgment. The direction and force of the wind in the next two columns; then the weather indicated by certain letters, and the height of the barometer and thermometer, generally registered in ordinary weather every four hours. And in the last column everything connected with the working of the ship, making and shortening sail, bearing of lights, headlands, &c., and any extraordinary circumstances are noted.

SHIP NONSUCH ON A VOYAGE FROM ENGLAND TO GIBRALTER, MONDAY, MAY 20TH, 1895.

P.M.

Hours.	Knots.	Tenths.	Course by Standard Compass.	Leeway.	Wind		Weather Bar. Therm.	Remarks.
					Direction.	Force.		
1	4	5	S. ½ E.	½ pt.	S.W. by W.	3	b.c.	At noon the Lizard Point bore north by standard compass, distant 12 miles, ship's head being south. Variation, 19° W. Deviation.—Ship's head South, 4° E.; S. ½ E., 4° E.; S. by E., 6° E.; S. ½ W., 2° E.; S. by W., 0; W.N.W. 24° W.; W. by N., 23° W.; W. ¾ N., 23° W.; W. by N. ½ N., 24° W.
2	5	—				to		
3	5	8	S. by E	0	„	4		
4	6	2						
5	6	5						
6	6	2	South.	¼	S.W. by W.	„		
7	6	—						
8	5	—	S. ½ W.	¾	W. S. W.	„		
9	4	5						
10	4	2						11 p.m.—Tacked ship.
11	6	5	S. by W.	½	„	„		
12	8	—	W.N.W.	¼	S.W. by W.	„		Midnight.

A.M.

TUESDAY, MAY 21st, 1895.

1	8	2	W. ½ N.	¼	S.S.W.	4 to	b.c.m.	12.15 a.m.—Sighted the Ushant lights bearing W.S.W.
2	8	4				5		
3	8	2	W. by N.	½	S.W.	4		
4	7	8					,,	
5	7	—						
6	6	5	W. ¾ N.	¼	,,			
7	5	5			,,	3		
8	4	—	W. by N.	¼	,,		,,	
9	3	—				2		A current set the ship the last four hours S.E. (magc.) 1·5 knots per hour.
10	2	5				1 to		
11	2	—	W. by N. ½ N.	½	,,	2		
12	1	8			,,	1	,,	Noon.

Lat. D.R. 48° 14′ N.	Long. D.R., 60° 4′ W.	Co. and Dist. made good D.R., S. 18°, W. 109′.	True Bearing and Distance.
Lat Obs. ——	Long. Chron. ——	Current Co. and Dist. ——	——

At noon on May 20, 1895, the Lizard Point bore from the ship north by compass distant twelve miles. It is obvious, therefore, that the ship bore from the Lizard south, and if we know the latitude and longitude of the Lizard Point we can find the position of the ship. Let us take our departure, therefore, from the Lizard Point in Latitude 49° 58′ N., longitude 6° 11′ W., and for convenience we will tabulate the courses as on the opposite page, and apply the corrections in their proper order.

First we convert the compass course into degrees by the subjoined table, and apply the variation, deviation, and leeway, as directed at pp. 82 and 83.

NAMES OF POINTS.				Points	In Degrees.
NORTH.		SOUTH.			
					° ′ ″
N. ¼ E.	N. ¼ W.	S. ¼ E.	S. ¼ W.	0¼	2.48.45
N. ½ E.	N. ½ W.	S. ½ E.	S. ½ W.	0½	5.37.30
N. ¾ E.	N. ¾ W.	S. ¾ E.	S. ¾ W.	0¾	8.26.15
N. by E.	N. by W.	S. by E.	S. by W.	1	11.15. 0
N. by E. ¼ E.	N. by W. ¼ W.	S. by E ¼ E.	S. by W. ¼ W.	1¼	14. 3.45
N. by E. ½ E.	N. by W. ½ W.	S. by E. ½ E.	S. by W. ½ W.	1½	16.52.30
N. by E. ¾ E.	N. by W. ¾ W.	S. by E. ¾ E.	S. by W. ¾ W.	1¾	19.41.15
N.N.E.	N.N.W.	S.S.E.	S.S.W.	2	22.30. 0
N.N.E. ¼ E.	N.N.W. ¼ W.	S.S.E. ¼ E.	S.S.W. ¼ W.	2¼	25.18.45
N.N.E. ½ E.	N.N.W. ½ W.	S.S.E. ½ E.	S.S.W. ½ W.	2½	28. 7.30
N.N.E. ¾ E.	N.N.W. ¾ W.	S.S.E. ¾ E.	S.S.W. ¾ W.	2¾	30.56.15
N.E. by N.	N.W. by N.	S.E. by S.	S.W. by S.	3	33.45. 0
N.E. ¾ N.	N.W. ¾ N.	S.E. ¾ S.	S.W. ¾ S.	3¼	36.33.45
N.E. ½ N.	N.W. ½ N.	S.E. ½ S.	S.W. ½ S.	3½	39.22.30
N.E. ¼ N.	N.W. ¼ N.	S.E. ¼ S.	S.W. ¼ S.	3¾	42.11.15
N.E.	N.W.	S.E.	S.W.	4	45. 0. 0
N.E. ¼ E.	N.W. ¼ W.	S.E. ¼ E.	S.W. ¼ W.	4¼	47.48.45
N.E. ½ E.	N.W. ½ W.	S.E. ½ E.	S.W. ½ W.	4½	50.37.30
N.E. ¾ E.	N.W. ¾ W.	S.E. ¾ E.	S.W. ¾ W.	4¾	53.26.15
N.E. by E.	N.W. by W.	S.E. by E.	S.W. by W.	5	56.15. 0
N.E. by E. ¼ E.	N.W. by W. ¼ W.	S.E. by E. ¼ E.	S.W. by W. ¼ W.	5¼	59. 3.45
N.E. by E. ½ E.	N.W. by W. ½ W.	S.E. by E. ½ E.	S.W. by W. ½ W.	5½	61.52.30
N.E. by E. ¾ E.	N.W. by W. ¾ W.	S.E. by E. ¾ E.	S.W. by W. ¾ W.	5¾	64.41.15
E.N.E.	W.N.W.	E.S.E.	W.S.W.	6	67.30. 0
E. by N. ¾ N.	W. by N. ¾ N.	E. by S. ¾ S.	W. by S. ¾ S.	6¼	70.18.45
E. by N. ½ N.	W. by N. ½ N.	E. by S. ½ S.	W. by S. ½ S.	6½	73. 7.30
E. by N. ¼ N.	W. by N. ¼ N.	E. by S. ¼ S.	W. by S. ¼ S.	6¾	75.56.15
E. by N.	W. by N.	E. by S.	W. by S.	7	78.45. 0
E. ¾ N.	W. ¾ N.	E. ¾ S.	W. ¾ S.	7¼	81.33.45
E. ½ N.	W. ½ N.	E. ½ S.	W. ½ S.	7½	84.22.30
E. ¼ N.	W. ¼ N.	E. ¼ S.	W. ¼ S.	7¾	87.11.15

ANALYSIS OF DAY'S WORK.

Compass Course.	Deviation.	Magnetic Course.	Variation.	True Course.	Leeway	True Course made good.	Distance	Diff. Lat.		Departure.	
								N.	S.	E.	W.
South	4° E.	S. 4° W.	19° W.	S. 15° E.	—	S. 15° E.	12·0		11·6	3·1	
S. $\frac{1}{2}$ E.	4° E.	S. 1$\frac{1}{2}$° E.	,,	S. 20$\frac{1}{2}$° E.	5$\frac{1}{2}$° W.	S. 26° E.	9·5		8·5	4·2	
S. by E.	6° E.	S. 5° E.	,,	S. 24° E.	—	S. 24° E.	18·5		16·9	7·5	
South	4° E.	S. 4° W.	,,	S. 15° E.	3° W.	S. 18° E.	12·2		11·6	3·8	
S. $\frac{1}{2}$ W.	2° E.	S. 7$\frac{1}{2}$° W.	,,	S. 11$\frac{1}{2}$° E.	8$\frac{1}{2}$° W.	S. 20° E.	13·7		12·9	4·7	
S. by W.	—	S. by W.	,,	S. 8° E.	6° W.	S. 14° E.	6·5		6·3	1·6	
W.N.W.	24° W.	S. 88$\frac{1}{2}$° W.	,,	S. 69$\frac{1}{2}$° W.	3° E.	S. 72$\frac{1}{2}$° W.	8·0		2·4		7·7
W. $\frac{1}{2}$ N.	22$\frac{1}{2}$° W.	S. 73° W.	,,	S. 54° W.	2$\frac{1}{2}$° E.	S. 56$\frac{1}{2}$° W.	16·6		9·2		13·8
W. by N.	23° W.	S. 78° W.	,,	S. 59° W.	6° E.	S. 65° W.	23·0		9·7		20·8
W. $\frac{3}{4}$ N.	24° W.	S. 74$\frac{1}{2}$° W.	,,	S. 55$\frac{1}{2}$° W.	3° E.	S. 58$\frac{1}{2}$° W.	12·0		6·3		10·3
W. by N.	23° W.	S. 78° W.	,,	S. 59° W.	3° E.	S. 62° W.	9·5		4·5		8·4
W. by N. $\frac{1}{2}$ N.	24° W.	S. 83° W.	,,	S. 64° W.	6° E.	S. 70° W.	3·8		1·3		3·6
Current —	—	S 45° E.	,,	S. 64° E.	—	S. 64° E.	6·0		2·6	5·4	
									103·8 S.	30·3 E.	64·6 W. 30·3 E.
											34·3 W.

The above is an analysis of this day's work. The distance run on each course has been taken from the second and third columns of the day's work. Thus, on the first course S. ½ E., she sailed 4·5 knots in the first hour and 5 knots in the second, making a total of 9·5 knots on that course; and so on with all the other courses. We have taken out from the traverse table the difference of latitude and departure for each true course and distance run. It will be seen that we have made no northing whatever, all our difference of latitude has been made to the southward, but that when on the starboard tack we have made easting and on the port tack westing, the difference being 34·1 knots to the westward. We will now proceed to find the position of the ship by applying this diff. lat. and the diff. long. found by inspection from the departure by the traverse table entered with 49° Mid. lat. as a course and departure 34·3 in the diff. lat. column, the corresponding distance 52′ being the diff. long. required, to the latitude and longitude of the Lizard Point.

Latitude Lizard Point..	49° 58′ N.	Longitude Lizard Point	5° 12′ W.
Diff. lat.	1° 44′ S.	Diff. long................	52′ W.
Lat. in at noon on May 21, 1895	48° 14′ N.	Long. in at noon, May 21, 1895	6 4′ W.

To find course and distance made good by Dead Reckoning:

This is found by entering the traverse table with the diff. lat. 103′·8 and departure 34′·3 given by the day's work in their proper columns, the course and distance corresponding thereto being the course and distance sailed by dead reckoning.

Thus 103·8 in diff. lat. column and 34·3 in departure column, gives course S. 18° W. and distance 109 miles.

CURRENT SAILING.

When a ship sails from one place to another, between which places a current runs at a uniform rate and in a certain direction, we can find by calculation the course which the ship must steer to allow for such current. For example (Fig. 21), the true course from A to B is S. 25° E.,

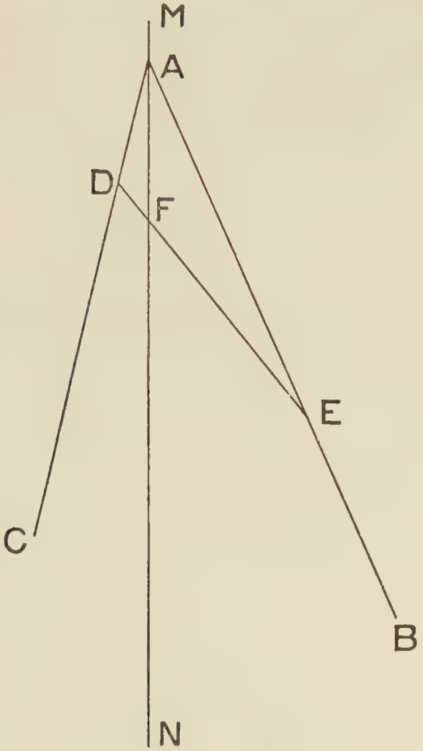

FIG. 21.

and a current runs S. 10° W. at the rate of 2·5 knots. Find what course must be steered to counteract the effect of this current, the ship's rate of sailing being 8·5 knots.

Let M N be the meridian of A, angle N A B, the course S. 25° E., A C the direction of the current, therefore angle C A N = 10°. On any convenient scale lay off from A on the line A C, A D = 2·5, open the dividers to 8·5 on the same scale and place one point on D, and find where the other point cuts the path which you wish the ship to take, A B at E; join D and this point E, then the line D E is the course the ship must steer. In one hour the current will set the ship 2·5 knots in the direction A C, or, in other words, will bring her to D, and in the same hour, if the ship be steered in the direction D E as she is going 8·5 knots, she will arrive at E.

Now in the plane triangle D A E, angle A = D A N + N A E = 10° + 25° = 35°, the side A D = 2·5 and the side D E = 8·5.

To find the course to be steered :

D E : D A :: sin. D A E : sin. D E A or
8·5 : 2·5 :: sin. 35° : sin. D E A

Log. sin. 35°... = 9·758591
Log. 2·5......... = 0·397940
10·156531
Log. 8·5......... = 0·929419
Log. sin. D E A = 9·227112
Angle D E A... = 9° 42′ 45″
Angle D A E... = 35° 0′ 0″
Sum......... = 44° 42′ 45″
180° 0′ 0″
Angle A D E... = 135° 17′ 15″

In the triangle A D F we have the two angles D A F

= 10° and A D F = A D E = 135° 17′ 15″; we can, therefore, find the remaining angle, thus:

$$\begin{aligned}\text{Angle A F D} &= 180° - \text{angle A D F} - \text{angle D A F}\\ &= 180° - 135°\ 17'\ 15'' - 10° = 34°\ 42'\ 45''\end{aligned}$$

but angle A F D = angle N F E, the course required. The course the ship must steer is, therefore, S. 34° 42′ 45″ E.

To find distance made good:

Sin. D A E : sin. A D E :: D E : A E or
Sin. 35° : sin. 135° 17′ 15″ :: 8·5 : A E

$$\begin{array}{lr} \text{Log. D E 8·5} \ldots\ldots\ldots\ldots = & 0·929419 \\ \text{Log. sin. A D E 135° 17′ 15″} = & \underline{9·847295} \\ & 10·776714 \\ \text{Log. sin. D A E 35°} \ldots\ldots = & \underline{9·758591} \\ \text{Log. A E} \ldots\ldots\ldots\ldots\ldots\ldots = & \underline{1·018123} \end{array}$$

A E = 10·43

and the distance she will make good in each hour will be 10·43 knots.

This problem is referred to again in the chapter on charts.

CHAPTER XII.

THE SEXTANT.

THE instrument commonly used in navigation and nautical astronomy for measuring angles is the sextant.

Fig. 22 is the representation of a sextant.

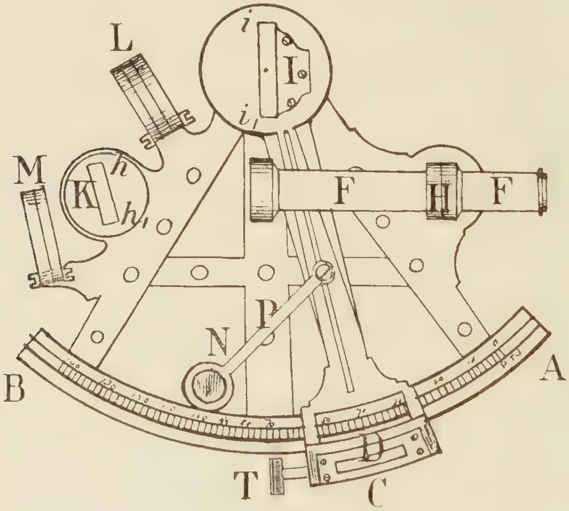

FIG. 22.

A B is the arc, which is divided into degrees and ten minutes, and D is the vernier, which is divided into minutes and ten, fifteen or twenty seconds of arc; the finer the division the better the instrument. The arc of a sextant, though only subtending a little more than 60° from the centre (I) is divided into double that number of degrees, the reason for this being given in the undermentioned proof of the sextant. N is a small magnifying glass on a pivoted bar (P) to facilitate reading the angles on the arc. I C is a movable arm, called the radius bar, pivoting at I, on which is fixed a mirror $i\,i_1$, perpendicular to the plane of the instrument, called the index glass, and the vernier on the other extremity at D. There is a clamp screw under the end of this arm near C to fasten it to the arc when taking an angle. T is called the tangent screw, and by turning it the vernier is moved very gradually over the arc after the arm has been clamped; $h\,h_1$ is a piece of glass fixed perpendicularly to the plane of the instrument, the half nearest to the frame being quicksilvered and the remainder left transparent. It is called the horizon glass. L and M are coloured shades of different opacities. H is the collar into which the telescope F is screwed.

In Fig. 23 let A B be the arc of the sextant, whose centre is I, with its zero at A. Let $i\,i_1$ be the index glass, $h\,h_1$ the horizon glass, and I C the radius bar. Suppose a ray of light to come from a heavenly body in the line X O, impinging on the surface of the index glass at I and being reflected by it in the line I K, and again reflected by the horizon glass in the direction K O. If the eye of an observer be at O it is evident that the reflection of the heavenly body and the horizon, as seen through the unsilvered part of the horizon glass which we will suppose to be in the direction O H, will appear to be in one.

Make K D parallel to I A, A being zero on the arc, then the arc C A, measured by the angle C I A, is one-half the

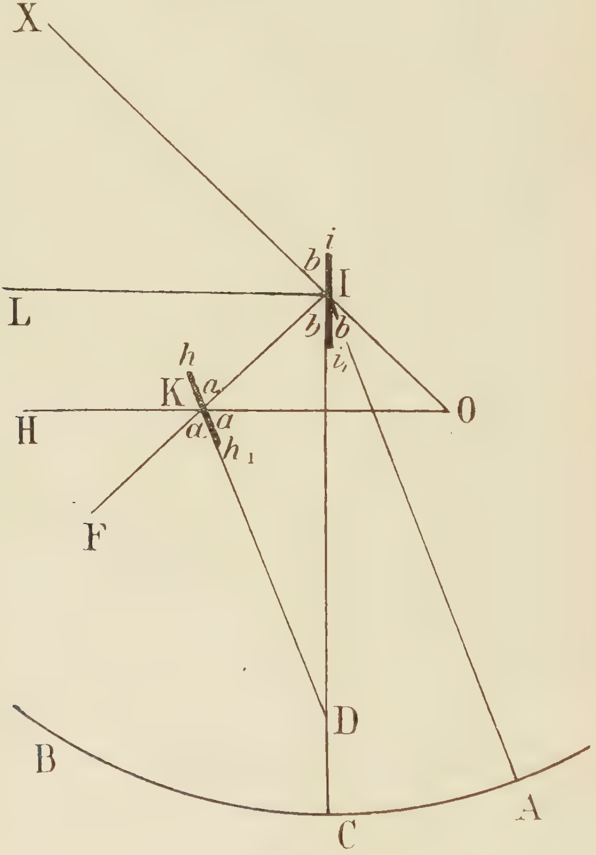

FIG. 23.

angle X O H, which is the altitude of the heavenly body. Here follows the proof of this statement.

By a well known law of optics the angle of incidence is equal to the angle of refraction. Therefore, in Fig. 23, the angle X I i equals the angle K I D; and the angle h K I equals the angle h_1 K O, because they are the complements of the angles of incidence and refraction. But the angles X I i and h K I are equal respectively to their opposite angles D I O and F K D (Euclid, Book I., Prop. 15). Therefore the angles marked *a* in the figure are equal to one another, as also are the angles marked b.

Now in the triangle K I O the two interior and opposite angles K I O and I O K are equal to the exterior angle F K O (Euclid I., 32).

$$\begin{aligned} FKO &= KIO + IOK \\ \text{or } 2a &= 2b + IOK \\ \therefore\ a &= b + \tfrac{1}{2}\, IOK \qquad (1) \end{aligned}$$

and in the triangle K D I

$$\begin{aligned} \text{the exterior angle } a &= b + KDI \qquad (2) \\ \therefore b + \tfrac{1}{2}\, IOK &= b + KDI \\ \text{or } \tfrac{1}{2}\, IOK &= KDI \text{ or } DIA \end{aligned}$$

since K D is parallel to I A.

But I O K is the altitude of the heavenly body, and D I A is the angle which measures the arc A C, therefore

$$\tfrac{1}{2} \text{ altitude} = \text{arc } AC.$$

It is therefore necessary to cut the arc of the sextant into double the number of degrees due to its angular value in order that the reading off of the angles observed may be correct.

Adjustments of the Sextant.

The first adjustment of the sextant is to make the index glass perpendicular to the plane of the instrument. To do this we proceed as follows: (1) Clamp the radius bar of the sextant at about the centre of the arc. (2) Look into the index glass with the arc turned away from you; if the reflection of the arc in the index glass is exactly in a line with the arc itself the adjustment is perfect; if not, the screw at the back of the index glass must be turned until the adjustment is made.

The second adjustment is to make the horizon glass perpendicular to the plane of the instrument; thus (1) Clamp the radius bar with the arrow on the vernier pointing to zero on the arc. (2) Look through the telescope F (Fig. 22), screwed into the collar H (Fig. 22), and by moving the tangent screw cause the reflected image of the sun or any other well-defined object to pass over the sun or such object seen through the horizon glass; if they exactly cover one another the adjustment is perfect, but if they do not, a screw at the side or back of the horizon glass must be turned until the adjustment is made. Or we can look at the horizon holding the sextant obliquely when the reflected horizon appears as one unbroken line with the horizon seen through the horizon glass.

The third adjustment is to make the index and horizon glasses parallel to each other when the arrow on the vernier is set to zero on the arc. This adjustment is made by turning a screw on the horizon glass till the reflected image of any object exactly coincides with such object seen through the horizon glass, the index bar being clamped at zero on the arc.

I would, however, strongly advise my readers to avoid touching the adjusting screws; better send the sextant to an expert if much out of order. When the error is small, it is, perhaps, well to leave it unadjusted, and to find the amount thereof, which must be applied to all angles observed with the instrument; this is called the index error. It can be found either by the horizon or by the sun.

(1) By the horizon. With the sextant perpendicular bring the reflected horizon exactly in line with the true horizon seen through the horizon glass, and read off the angle which will be the index error required, + if off the arc, or to the right of zero, and – if on the arc, or to the left of zero.

(2) By the sun. Screw the inverting telescope F F (Fig. 22), into the collar (H), and set the radius bar (C I) nearly at zero (A). With the shades I and M cause the sun and his reflection to be bearable to the eye, and then by moving the tangent screw (T) round, make the edge of the reflected image of the sun just touch the real sun: read off the angle which we will suppose to be 33′ 40″. Next, by turning the tangent screw the reverse way, make the reflected sun pass over the real sun, so that they touch again on the other side. We now get an angle off the arc to the right of zero; let us suppose this to be 31′ 50″ off, and we proceed to find the index error thus:

Reading on the arc 33′ 40″
Reading off the arc 31′ 50″

Difference 2) 1′ 50″ on the arc

∴ Index error = 0′ 55″ minus, that is to be subtracted from the altitude or angle observed.

If the reading off the arc were the greater, the index error would be plus, that is to be added to the altitude or angle observed. A ready method of proving the accuracy of the observation is to add together the readings on and off the arc; one-fourth of this total should equal the semi-diameter of the sun given in the Nautical Almanac for the date of observation. Thus

Reading on the arc	33′ 40″
Reading off the arc	31′ 50″
	4)65′ 30″
Semi-diameter of the sun on the date of observation	16′ 22″

USE OF THE SEXTANT.

The sextant is used by the navigator to take the altitudes of the sun, moon, and other heavenly bodies above the visible horizon. It is useful in conjunction with the azimuth compass to fix the position of the ship when near land. With it we can measure the angular distance between headlands, and thus keep our vessel clear of sunken dangers, as we shall see hereafter. It is also indispensable in men-of-war for keeping station when sailing in company, and during nautical evolutions. It affords an easy means of knowing whether we are gaining on a vessel ahead or not. And, in numberless other ways, is invaluable to the seaman.

To observe an altitude of the sun, take the sextant by the handle in the right hand, turn the shades (L, Fig. 22) back, unclamp the radius bar and hold it in the left hand at about zero, look towards the sun through the collar and horizon glass with the arc downwards, and you will see the reflected sun in the right hand part of the horizon glass. Move the radius bar slowly from you, and follow the reflected sun down till it is about on a line with the horizon, turn the sextant horizontal and clamp the radius bar, screw in a telescope (the inverting telescope gives the best results, but is more difficult to observe with), having focussed it to suit your sight, and draw forward one or more of the shades (L). Now look at the horizon under the sun, and with the tangent screw cause the reflected solar image just to touch the horizon, taking care to pivot the sextant in your hand while observing, with the tube as a centre, so as to ensure that you are getting the perpendicular altitude, otherwise the angle will be too large.

As a general rule, when observing the sun the inverting telescope is used, In Fig. 24 we have a representation of the sun (*a*) as seen through the inverting tube, when an altitude of its lower limb is observed, the horizon being HH_1, and the sea above it; (*b*) is the sun similarly seen when the upper limb is observed.

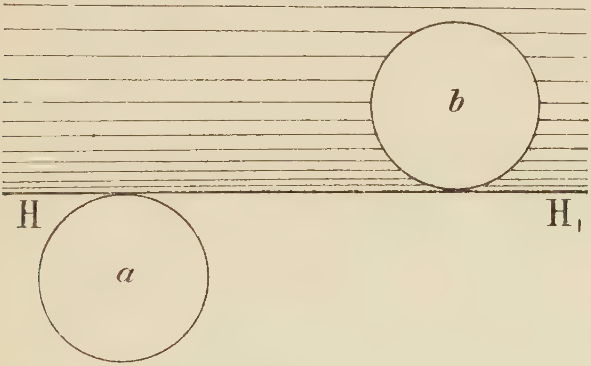

FIG. 24.

It is almost needless to add that the sun is immeasurably the easiest and best heavenly body to observe.

Altitudes of the moon, and of the more brilliant planets and stars, however, at about sunset, when the horizon is distinct, are very good, but the upper or lower limb of the moon must be perfect so that an accurate contact with the horizon may be made.

In taking angles between headlands or any other objects, the reflected image of one of them in the horizon glass is made to coincide with the other object seen through the transparent part of the horizon glass.

It is almost impossible to explain in writing how to read off the angles measured by the sextant. Where the arrow on the vernier cuts the arc it shows the degrees and ten minutes of the angle, and where a line on the vernier and one on the arc coincide it gives the remaining minutes and ten seconds on the vernier. Thus, suppose the arrow cuts between 72° 10′ and 72° 20′, the angle is greater than the former and less than the latter ; then, suppose any line on the arc is exactly in the same straight line as 7′ 20″ on the vernier, the reading off will be 72° 17′ 20″. But a short practical examination of the instrument will be found more useful than a volume of description.

CHAPTER XIII.

ON TIME.

TIME is measured by the period which elapses between the recurrence of certain celestial phenomena. For instance, the intervals between the successive arrivals of the sun at the meridian of an observer owing to the daily rotation of the earth are called apparent solar days. The angle between the ecliptic, which is the apparent path of the sun in the heavens in the course of a year, due to the earth's revolution round that body, and the equinoctial or celestial equator, and also the fact that the earth's distance from the sun is constantly changing during the year because her orbit is slightly elliptical, and her velocity varies inversely as her distance from the sun—in other words, she moves fastest when she is nearest to that luminary—cause successive apparent solar days to be unequal in length. It is impossible to construct an instrument to measure accurately the time given by transits of the real, or, as it is termed, the apparent sun over the meridian of an observer; therefore it has been found convenient to suppose that there is an imaginary body, called the mean sun, which, starting with the apparent sun from the first point of Aries and moving at a uniform rate along the equinoctial, returns again to that point at the same moment as the apparent sun. From December 23 to April 15 the true sun is in advance of the mean sun; from April 15 to June 14 the true sun is behind the mean sun; from June 14 to August 31 the true sun is in advance of the mean sun; and from August 31 to December 23 the true sun is behind the mean sun.

Clocks and watches are necessarily regulated to mean time—that is, time measured by the motion of the mean sun; they will, therefore, be fast of apparent time from December 23 to April 15, slow from April 15 to June 14, &c.

Let Fig. 25 be a diagram on the plane of the horizon, N W S E the horizon, P the pole, W Q E the equinoctial, N S the meridian of the observer, Z the zenith, A the first point of Aries, A C the ecliptic, X the apparent sun whose declination is X R and right ascension A R, and m the mean sun whose declination is zero, as he is always on the equator, and his right ascension A m. The *apparent time* of the observer, whose zenith is Z, is the polar angle X P Z, or the arc Q R. His *mean time* is the polar angle m P Z or arc m Q, and his *sidereal time* or the *right ascension of his meridian*, the polar angle A P Z or arc A Q. The polar angle X P m or the arc R m is the equation of time—that is, the difference between the times given by the apparent and mean suns. It is found in the Nautical Almanac on pages I. and II. in each month, and is used for converting mean time into apparent time, and *vice versâ*.

A *sidereal year* is the interval between the sun's leaving a fixed star and his return to it, owing to the revolution of the earth round that body, or the time between two successive conjunctions of the sun with a fixed star as seen from the earth.

A *tropical or mean solar year* is the time which elapses between the sun leaving the first point of Aries and his return to it.

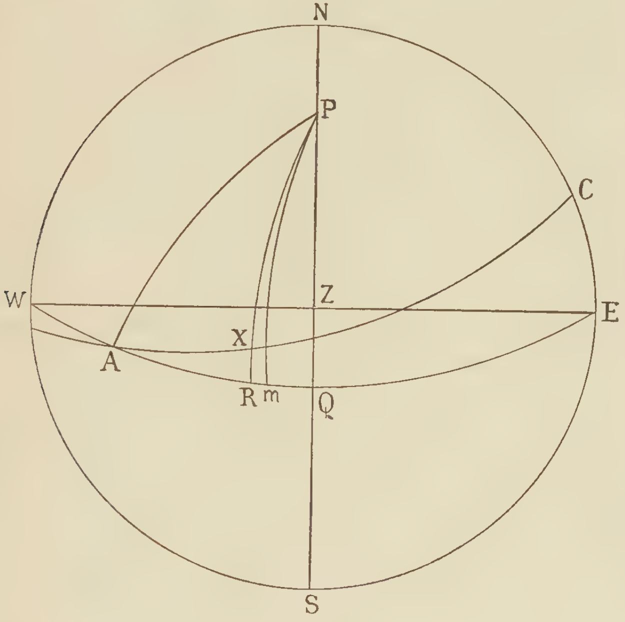

FIG. 25.

The mean sun in twenty-four hours makes the complete circle of the equator—that is, 360° in arc or longitude—so that in one hour he will travel over 15°, in two hours 30°, in twelve hours 180°, and so on. Let us suppose him to be over the meridian of Greenwich. The mean time at Greenwich will be noon, and it will be the same time at all places situated on the same meridian. One hour afterwards the mean sun will have moved, owing to the earth's rotation on her axis from west to east, to 15° west longitude. It will now be 1 p.m. mean time at Greenwich, or, as we shall write it for greater facility, 1 p.m. G.M.T. (Greenwich mean time), at 3 p.m. G.M.T. the mean sun will be over 45° west longitude; at 5 p.m. G.M.T. over 75° west longitude; at midnight G.M.T. over 180° west or east longitude.

It is also clear that at 10 a.m. G.M.T. the mean sun will be over 30° east longitude, because the mean sun in two hours will move over 30° of longitude; at 11 a.m. G.M.T. over 15° east longitude, &c. In each case there is one hour difference in mean time for each 15° of longitude. Of course, there would be thirty minutes of time for 7° 30′ of longitude, four minutes of time for 1° of longitude, four seconds of time for 1′ of longitude, and so on.

Let me digress for a moment to explain that an astronomical day commences and ends at noon, and not at midnight as does a civil day, and that there is no a.m. or p.m. time; the hours run from no hours to twenty-four hours from noon to noon. Thus, noon on January 28 civil time is 0h. 0m. 0s. (no hours, no minutes, no seconds) on the same date, or 24h. 0m. 0s. on January 27 astronomical time. 3 p.m. on the 28th civil time is 3h. 0m. 0s. on the 28th astronomical time. 9 a.m. on the 28th is 21h. 0m. 0s. on the 27th astronomical time. Midnight on the 9th–10th is 12h. 0m. 0s. on the 9th astronomical time, &c. It is very important that the reader should understand how to find astronomical from civil time, because in all references to the Nautical Almanac the former time is employed. I would also advise navigators invariably to make a distinction between time and arc by writing *h.*, *m.*, and *s.* for hours, minutes, and seconds of time, so that they may never be confounded with the minutes and seconds (′, ″) of arc; otherwise great confusion and error will probably arise.

We have seen when it is noon, mean time, at Greenwich, as the mean sun travels from east to west, it must have been on the meridian of a place in 15° E. longitude one hour previously, and at such a place the mean time will now be 1 p.m. Also that one hour will elapse before mean noon at a place in 15° west longitude; therefore the mean time at that place will be 11 a.m. It is evident, therefore, that if we know the mean time at Greenwich, and can find the mean time at the ship at the same instant, the difference between these two times will give us the time which the mean sun takes in going from the meridian of Greenwich to the meridian of the ship, and that is the longitude in time. On this fact are founded very nearly all problems for finding the longitude.

ON THE CORRECTIONS TO BE APPLIED TO THE OBSERVED ALTITUDE OF A HEAVENLY BODY IN ORDER TO FIND THE TRUE ALTITUDE.

The true altitude of a heavenly body is the arc of a circle of altitude intercepted between the rational horizon and the centre of the body, and it is measured by an angle at the centre of the earth—that is, the centre of the celestial globe.

It is obvious that, as the semi-diameter of the earth is about 4000 miles, a correction must be made to reduce the altitude observed at a position on its surface to that which it would be if taken from its centre. This correction is called *parallax*. It is very great with regard to the moon, which is only about 240,000 miles distant; and must also be allowed for in correcting the observed altitude of the sun and that of the nearer planets. But the fixed stars are at such enormous distances from the earth that, with very few exceptions, the whole diameter of the earth's orbit—a distance of about 180,000,000 miles—makes no difference in their apparent position in the heavens. Such an insignificant distance as the earth's semi-diameter is therefore inappreciable in this regard, and the parallax of such bodies being under one second of arc (1″) may be neglected for our work.

The rational horizon is a great circle whose plane passes through the centre of the earth; it is equi-distant from the zenith and nadir.

The sensible horizon is a circle whose plane is parallel to the rational horizon, touching the surface of the earth immediately below the observer. This may be generally considered as a great circle, the diameter of the earth being such an infinitesimal quantity when compared with the celestial globe.

The visible horizon is the circle which bounds the view of the sea, as seen by an observer at the centre.

On board ship, when taking the altitude of any heavenly body, an observer is raised above the level of the sea, and the visible horizon will be lower than the sensible horizon. A correction called the *dip* has, therefore, to be subtracted from the observed altitude.

There is another correction called *refraction,* which is due to the rays of the light from any heavenly body being deflected from a straight line in passing through the earth's atmosphere. This is greatest at low altitudes, and gradually disappears as the heavenly body nears the zenith. It is always subtractive from the observed altitude, as its effect is to make the body appear higher than it actually is.

In observing the altitude of the sun, moon, and some of the larger planets, the angular measurement of either the lower or upper edge of the apparent disc of such bodies to the visible horizon is taken by the sextant, the upper and lower edges being called the upper and lower limbs respectively. It then becomes necessary to take the *semi-diameter* of such body out of the Nautical Almanac and add it to the altitude if the lower limb has been observed, or subtract it if the upper, in order to find the altitude of the centre.

In Fig. 26 let A D E be the terrestrial globe, Z H H_2 a portion of a circle of altitude, and X a heavenly body. Then the angle X O H or the arc S H is the true altitude of such body at A on the surface of the earth. H is the point when the rational horizon cuts the circle of altitude, H_1 where the sensible horizon cuts it, and H_2 where the visible horizon cuts it from an observer whose height above the surface of the earth is A B. The angle X B H_2 is the observed altitude of X, and X A H_1 is the apparent altitude.

The correction to be applied to the altitude observed at B, to reduce it to what it would be if observed at A, is called the *dip,* and is always subtracted from the former, the angle X B H_2 being obviously greater than the angle X A H_1.

The angle A X O or S X T, or the arc S T is the parallax in altitude, which is always additive to the apparent altitude, as it is evident from Fig. 26 that the place of a heavenly body in the celestial concave as seen from the surface of the earth at A is lower than it would appear if seen from the centre of the earth at O.

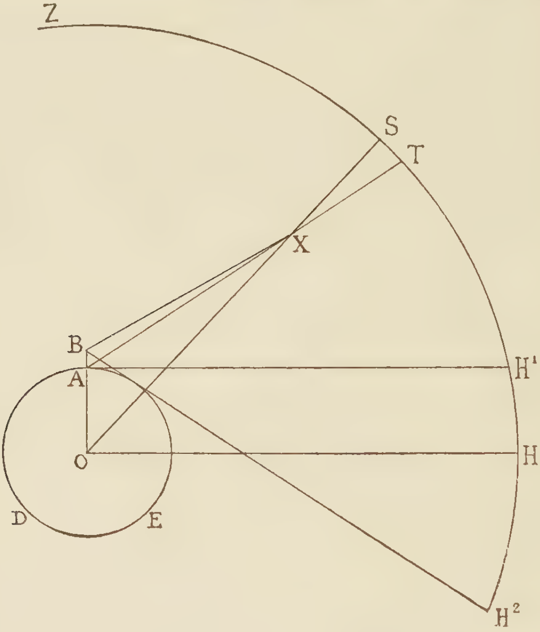

FIG. 26.

We see, then, that to find the apparent altitude of a heavenly body from that observed, it is necessary to apply the following corrections: Dip, subtractive; and semi-diameter, additive if lower limb was observed, subtractive if upper limb. Having the apparent altitude, which is the angle X A H_1 we find the true altitude by subtracting the refraction and adding the parallax, arc S T, to obtain the arc S H or the angle S O H, which is the true altitude required.

CHAPTER XIV.

On Right Ascension and Time.

We have seen that the Right Ascension of a heavenly body is the arc of the equinoctial intercepted between the first point of Aries and the meridian of such body measured eastward from the former. Now the mean time at any place on the earth's surface is the angle at the pole between the meridian of such place, measured to the westward, and the meridian passing through the mean sun, or the arc of the equinoctial, intercepted between those meridians. Also the hour angle of any heavenly body is the angle at the pole between the meridian of the observer and that passing through the centre of the body measured in the same manner as in mean time, that is to the westward from the meridian of the observer. The hour angle of the real or apparent sun is apparent time at ship (S.A.T). The hour angle of the mean sun is the mean time of the observer or ship mean time (S.M.T.) The polar angle between the meridians of Greenwich and that passing through the mean sun is mean time at Greenwich or (G.M.T.).

Fig. 27 is constructed on the plane of the equinoctial. Let P be the north pole and Q T B S the equinoctial. Now we will suppose A to be the first point of Aries, in the mean sun, and X the apparent sun. The angle A P m or the arc A m is the right ascension of the mean sun (R.A.M.S.). Now let P B be part of the meridian of the observer; then the angle m P B or the arc B m is the ship mean time (S.M.T.) Let P G be part of the meridian of Greenwich; then the angle m P G or the arc G m is the mean time at Greenwich or (G.M.T.). The angle X P B or the arc B R is the apparent time of the observer or ship apparent time (S.A.T.), and the angle m P R or the arc m R is the equation of time.

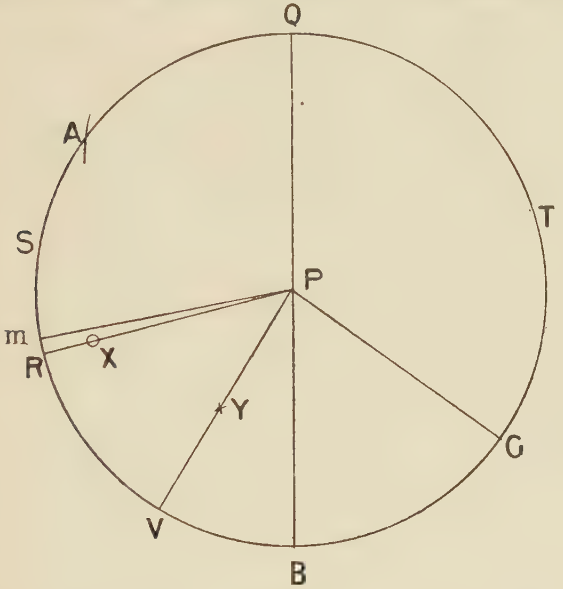

FIG. 27.

It is very important to understand that right ascension, though given in hours, minutes, and seconds, is not time. As we have already seen, it is a means of defining the position of a heavenly body, with regard to the first point of Aries. It is, however, used in the solution of problems involving time and longitude thus: If in Fig. 27 we suppose Y to be the position of a star, and we ascertain the star's hour angle (H.A.), we are able to find the mean time of the observer (S.M.T.) in this manner. We take out from the Nautical Almanac the right ascension of the star, which is the arc A V or the polar angle A P V, and we add to it the star's hour angle, the arc B V or polar angle V P B, and so obtain the arc A B or the polar angle A P B which is the right ascension of the meridian of the observer. We next deduct the right ascension of the mean sun, the arc A m, and so obtain the arc B m, or the

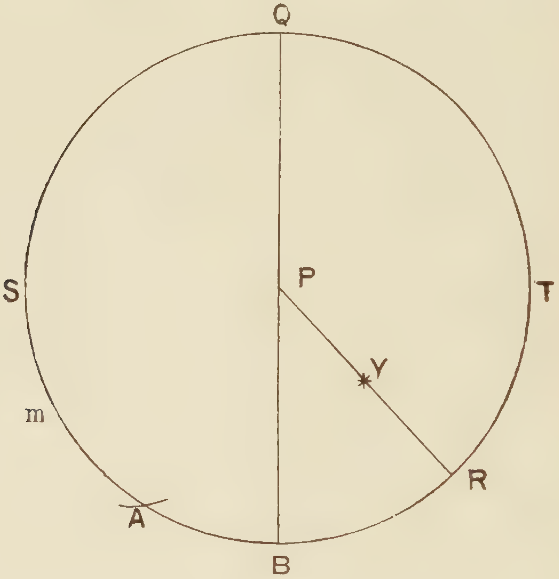

FIG. 28.

mean time of the observer. And thus we establish the equation

$$\text{R.A.M.S.} + \text{S.M.T.} = \text{R.A. Star} + \text{H.A. Star.}$$

and, having any three of these, we are able to find the fourth.

But sometimes the arc thus found exceeds 24 hours, in which case it must be diminished by 24 hours. Or the sum of the two arcs on one side is less than the one arc on the other, then 24 hours must be added to the sum of the first two, and the latter subtracted from this amount to find the fourth term.

In Fig. 28, let A be the first point of Aries, m the mean sun, P B the meridian of the observer, and P R the meridian of a star or planet, or that of the moon. Then the arc A T Q m is the right ascension of the mean sun; B m the ship mean time; B S Q T R, the hour angle of star; and A R, the star's right ascension. Therefore,

$$\text{arcs } \mathrm{ATQm} + \mathrm{Bm} = \mathrm{BSQTR} + \mathrm{AR}$$

and the figure shows that this equation is correct.

ON THE NAUTICAL ALMANAC.

All the data in the Nautical Almanac are calculated for Greenwich time, either mean or apparent. Thus on page I. in each month we find the right ascension and declination of the apparent sun, and the equation of time at apparent noon at Greenwich. On page II. we have the same data at mean noon at Greenwich, and in addition we are given the sun's semi-diameter, which may be assumed to be the same as at apparent noon (see foot-note in the Nautical Almanac), and the sidereal time or right ascension of the meridian of Greenwich, when it is mean noon at that place. In the columns following the right ascension and declination on page I. we have their respective variations in one hour. When employing any data to work out problems in nautical astronomy it is necessary to find the time at Greenwich when the observation was taken, and make the allowance due to the time elapsed since noon at Greenwich.

To Find Greenwich Time.

The first thing to be done in working out a problem in nautical astronomy is to find the approximate time at Greenwich when the observation was taken. To do this we require to know the longitude nearly and the time at the place of the observer. For example, an observation is taken at 9 a.m. on March 25 in longitude 27° 10′ W. As we have already seen, 9 a.m. on March 25 is equal to 21h. 0m. on March 24, astronomical time. We have next to turn the longitude to time. Table 26, page 112, Inman, and Tables A and B, page 1*, Norie, enable us to do this by inspection; but the following simple calculations make us independent of these tables.

To Convert Arc into Time.

Rule.—Multiply the arc given by 4, the minutes of arc become seconds, and the degrees become minutes of time, which divided by 60 become hours and minutes.

Examples.

(1) Turn 27° 10′ 0″ into time.

$$
\begin{array}{lrrr}
 & 27^\circ & 10' & 0'' \\
 & & & 4 \\ \hline
60) & 108 & 40 & 0 \\ \hline
 & 1\text{h.} & 48\text{m.} & 40\text{s.} \\ \hline
\end{array}
$$

(2) Turn 111° 14′ 40″ into time.

$$\begin{array}{r} 111^\circ\ 14'\ 40'' \\ 4 \\ \hline 60)444\ \ 58\ \ 40 \\ \hline \underline{7\text{h}.\ 24\text{m}.\ 58\tfrac{4}{6}\text{s}.} \end{array}$$

To Convert Time into Arc.

Rule.—Multiply the time given by 3, and this result by 5.

Examples.

(1) Turn 1h. 48m. 40s. into arc.

$$\begin{array}{rrr} 1\text{h}. & 48\text{m}. & 40\text{s}. \\ & & 3 \\ \hline 5 & 26 & 0 \\ & & 5 \\ \hline \underline{27^\circ} & \underline{10'} & \underline{0''} \end{array}$$

(2) Turn 11h. 2m. 5s. into arc.

$$\begin{array}{rrr} 11\text{h}. & 2\text{m}. & 5\text{s}. \\ & & 3 \\ \hline 33 & 6 & 15 \\ & & 5 \\ \hline \underline{165^\circ} & \underline{31'} & \underline{15''} \end{array}$$

Nothing can be more easy than these calculations.

Well, then, on March 25 we find

that the ship time is .... 21h. 0m. 0s. on March 24;
the longitude in time is (West) ...... 1h. 48m. 40s.
And the Greenwich time is therefore 22h. 48m. 40s. on March 24.

We add the longitude in time to the ship time because the observer is in west longitude, and the time at Greenwich is therefore the greater.

If the observer is in east longitude his time will be greater than Greenwich time, and the longitude in time will therefore have to be subtracted.

The following verses may serve as an *aide-memoire:*

Greenwich time least, longitude east.
Greenwich time best, longitude west.

It may be well to give a few more examples. Find the Greenwich time corresponding to the following:

(1) 3 p.m. in longitude 110° E. on March 1, 1895.
(2) 1 a.m. ,, ,, 160° W. on May 3, ,,
(3) 11.10 p.m. ,, ,, 47°.10′ W. on June 14, ,,
(4) 10.20 a.m. ,, ,, 64°.11′ W. on Jan. 5, ,,
(5) 3.30 p.m. ,, ,, 95°.12′ E. on July 12, ,,

(1)

Ship time .. .. 3h. 0m. on March 1.
Longitude in time......... 7h. 20m.—
Greenwich time........... 19h. 40m. on Feb. 28.

Longitude = 110° E.
4
60)440 0 0
Longitude in time (East) 7h. 20m. 0s.

In this case we add 24 hours to the ship time and put the date one back; we then subtract the longitude in time because it is east.

(2)

Ship time .... 13h. 0m. on May 2.
Longitude in time (West).. 10h. 40m.+
Greenwich time .... 23h 40m. on May 2.

Longitude = 160° W.
4
60)640
Longitude in time = 10h. 40m.

Here we turn 1 a.m. into astronomical time, and add the longitude in time because it is west.

The Greenwich time for (3) is 14h. 18m. 40s. on June 14.
„ „ „ „ (4) is 2h. 36m. 44s. „ Jan. 5.
„ „ „ „ (5) is 21h. 9m. 12s. „ July 11.

which my readers would do well to verify for themselves.

TO APPLY THE CORRECTIONS TO RIGHT ASCENSION, DECLINATION, EQUATION OF TIME AS TAKEN FROM THE NAUTICAL ALMANAC.

After finding the time at Greenwich, we proceed thus: We take out from the Nautical Almanac the right ascension, declination, or whatever it is we want to find, and also its variation for one hour in its proper column, and we multiply this variation by the number of hours and fractions of an hour which have elapsed since that noon at Greenwich, for which we have taken out these data.

Let us suppose we want to find the declination of the sun at a place in 40° 12′ W. longitude, where an observation was taken at 8h. 30m. a.m., S.A.T., on Aug. 16, 1895.

We must first find the time at Greenwich:

Ship time............... 20h. 30m. 0s. Aug. 15.
Long. in time (West) 2h. 40m. 48s. +
Greenwich time...... 23h. 10m. 48s. on Aug. 15.

Longitude .... 40° 12′ W.
4
60)160 48
Longitude in time = 2h. 40m. 48s.

We next turn to the month of August in the Nautical Almanac, and take out the declination of the sun at apparent noon, page I., for Aug. 15. It is 14° 3′ 55″·4, and we note that it is *decreasing* in amount from day to day. From the next column we take the variation in one hour, viz., 46″·86. Now, as 23h. 10m. 48s. have elapsed since noon of the 15th, we must multiply this hourly difference by that number of hours and the fraction due to 10m. 48s. For practical purposes ·2 will be quite accurate enough. We proceed in this way:

Variation in one hour	46″·86
Time elapsed	23·2 hours
	9372
	14058
	9372
	60) 1087·152
Variation in 23·2 hours ...	18′ 7″·152

Sun's declination at Greenwich apparent noon on Aug. 15	14° 3′ 55″·4 N.
Variation in 23·2 hours	−18′ 7″
Sun's declination at the time of observation	13° 45′ 48″·4 N.

In this case *the variation is subtracted* because the sun's declination is *decreasing* from day to day.

The right ascension of the apparent sun at apparent noon at Greenwich is given on p. I., and at mean noon on p. II. It is corrected in exactly the same way as the declination. This right ascension, however, we have no occasion to use.

Let us now correct the equation of time for an observation taken at 4.30 p.m., S.A.T., in longitude 110° 5′ E. on Oct. 12, 1895.

		Longitude.........	110° 5′ E.
Ship time...............	4h. 30m. Oct. 12.		4
Long. in time (East)	7h. 20m. 20s., —		60)440 20
Greenwich time......	21h. 9m. 40s. Oct. 11.	Long. in time	7h. 20m. 20s.

Equation of time at Greenwich, apparent noon on Oct. 11	13m. 11·91s.
Variation in one hour, ·637s. × 21·2 hours.....................	+ 13·5s.
Equation of time at time of observation	13m. 25·41s.

We *add the variation* because we see in the Nautical Almanac that the equation of time is *increasing* from day to day.

Apparent time is generally kept on board ship, and that is why, following the usual custom, we have taken out these data for apparent noon. Should chronometer time be used, they would have to be looked for under mean noon and corrected in a precisely similar manner. The variation for one hour is always taken out under apparent noon.

CHAPTER XV.

ON FINDING THE LATITUDE BY OBSERVATION.—BY THE MERIDIAN ALTITUDE OF THE SUN, AND STAR ABOVE THE POLE.

WHEN the sun, or any other heavenly body above the pole, is on the meridian of a place he attains his greatest altitude, as the following figure demonstrates.

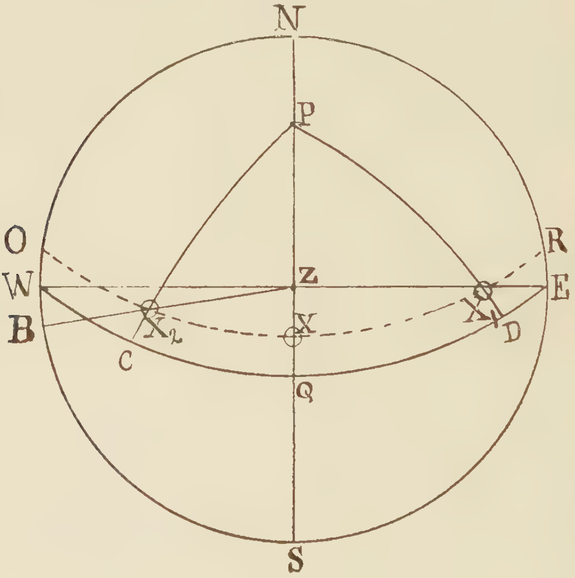

FIG. 29.

In Fig. 29 let N W S E be an astronomical diagram on the plane of the horizon; N S the meridian of an observer whose zenith is Z; P the north pole.

W Q E the celestial equator; P C and P D celestial meridians; X the sun on the meridian of the observer; X_1 and X_2 the sun in two positions when not on the meridian Z Q the latitude; Z X the zenith distance of the sun on the meridian, the zenith being north of that body; X Q, X_1 D, X_2 C, the sun's declination. As the earth rotates on its axis the sun will appear to move along the circle R X O, rising at R and setting at O; if his declination is increasing R E will be a few minutes of arc less than W O, and, if decreasing, R E will be a little greater than W O, but the increase or decrease cannot be shown in so small a figure as the above. The meridian altitude of the sun is X S, and his altitudes when at X_1 and X_2 are X_1 E and X_2 B respectively; it is obvious from the figure that they are smaller than the meridian altitude X S. It is equally clear that wherever the sun may be while he is above the horizon on the line R O, his altitude will be less than when he is at X on the meridian.

To take the meridian altitude of the sun, go on deck with a sextant some quarter of an hour or twenty minutes before noon and observe the sun's altitude, then keep his lower limb touching the horizon by turning the tangent screw till he ceases to rise and begins to fall. The highest altitude thus obtained is the observed meridian altitude of the sun's lower limb. From this we find the true altitude X S by applying the necessary corrections (see Chapter XIII.), which, deducted from 90°, gives us Z X, the zenith distance, zenith in this case north of the sun; we next apply the declination X Q, and, as it is north, we add it to the zenith distance and obtain the latitude, Z Q.

In Fig. 30 we suppose the celestial globe to be cut in half on the plane of a meridian, Z H N. Let O be the centre; B E C D the terrestrial globe, B C being the poles, D E the equator, A the position of an observer on the

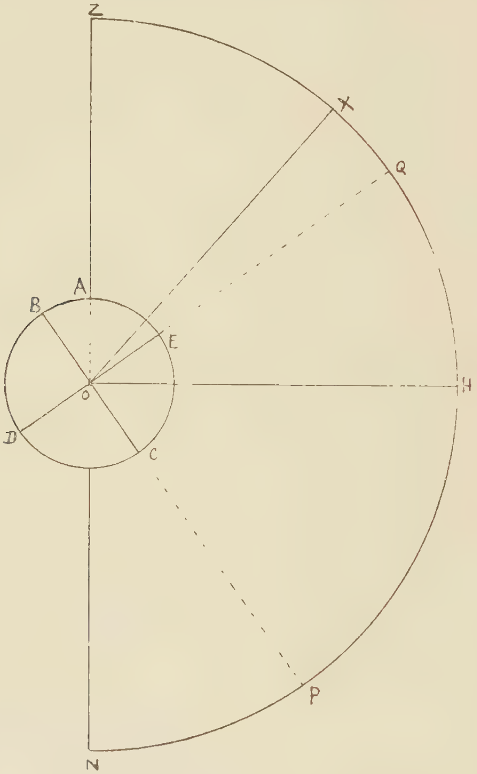

Fig. 30.

earth whose zenith is Z, whose nadir is N, and whose celestial meridian is Z H N; H is where the rational horizon cuts the meridian, and Q and P are the positions of the celestial equator and south pole respectively. Let X be the position of the sun on the meridian, then his meridian altitude is the angle X O H or the arc X H, his declination is Q X north, and his zenith distance is Z X, zenith north. Now the angle A O E or the arc A E is the latitude of A; but the angle A C E measures the arc Z Q, and therefore the arc Z Q = A E = the latitude of A. Having found Z X, the zenith distance, and X Q, the corrected declination, we find Z Q the latitude by adding them together—that is, because the zenith is north of the sun and the declination is also north.

When the zenith is north of the sun and his declination south, the latitude is found by taking the difference between those two arcs and naming it north or south, as the larger arc is north or south.

It is easy to see that when the sun on the meridian bears south from the observer, his zenith is north of the sun; or, if he is south of the sun, his zenith is also south. It is necessary to be clear about this, as the working out of the meridian altitude problem depends materially upon how the zenith is named.

EXAMPLES.

(1) On March 31, 1895, in longitude 24° 11′ W., the observed meridian altitude of the sun's lower limb was 41° 12′ 40″, sun bearing south; index error, + 1′ 20″; height of eye, 16ft.; required the latitude.

Declination at Greenwich Noon ...		4° 10′ 12″·4	N.
Corrn. for 24° W. long. *...........		1′ 33″	
Declination at Ship Noon...........		4° 11′ 45″	N.
Observed mer. alt......................	† ⊙	41° 12′ 40″	S.
I.E.	+	1′ 20″	
		41° 14′ 0″	
Correction ‡	+	11′ 6″	
True alt.		41° 25′ 6″	S.
		90° 0′ 0″	
Z.D.......................................		48° 34′ 54″	N.
Decn.		4° 11′ 45″	N.
Lat. in		52° 46′ 39″	N.

In the above example I have used tables 17 Inman and IX. Norie for correcting the altitude, and table XXI. Norie for correcting the declination. It is easy, however, to do this last by means of the longitude in time and the hourly variation from the "Nautical Almanac." Thus, 24° 11′ W. longitude equals 1h. 36m. 44s., multiplied by 57″·85, the hourly variation, gives us 1′ 32″·55 to be added to the declination at noon, because it is increasing, and the longitude is west. In the following examples it will be unnecessary to refer again to these tables.

(2) On June 15, in longitude 115° E., the observed meridian altitude of the sun's lower limb was 52° 10′ 30″, sun bearing north, index error, −2′ 30″; height of the eye, 17ft.; required the latitude.

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* Norie, XXI. † ⊙ means sun's lower limb.

‡ Inman, Table 17; Norie. Table IX.

Obs. mer. alt. ☉	52° 10′ 30″ N.	Declination............	23° 19′ 22″·6 N.
I.E. .... −	2′ 30″	7″·6 × 6·4 =	49″
	52° 8′ 0″	Decn. corrected	23° 18′ 33″ N.
Corrn.......... +	11′ 0″		
Tr. alt.	52° 19′ 0″ N.		
	90° 0′ 0″		
Zen. Dist. ...	37° 41′ 0″ S.		
Decn.	23° 18′ 33″ N.		
Lat.	14° 22′ 27″ S.		

(3) On Dec. 1, 1895, in longitude 170° W., the observed meridian altitude of the sun's lower limb was 76° 29′ 40″, sun bearing south; index error, − 1′ 50″; height of the eye, 24ft.; required the latitude.

Obs. mer. alt. ☉	76° 29′ 40″ S.	Declination.........	21° 49′ 30″·9
I. E............−	1′ 50″	23″·3 × 11·3 =	4′ 23″
	76° 27′ 50″	Decn. corrected...	21° 53′ 54″ S.
Corrn..........+	11′ 12″		
Tr. alt..........	76° 39′ 2″ S.		
	90° 0′ 0″		
Zen. Dist. ...	13° 20′ 58″ N.		
Decn.	21° 53′ 54″ S.		
Lat.	8° 32′ 56″ S.		

BY THE MERIDIAN ALTITUDE OF A STAR ABOVE THE POLE.

When a star is on the celestial meridian of an observer on the surface of the earth its right ascension is the right ascension of the meridian, which, as we have already seen (Chapter XIV.), is equal to the ship mean time + right ascension mean sun; that is, R.A. of star on the meridian = S.M.T. + R.A.M.S. To find the time of any star's meridian passage, therefore, we have to deduct the R.A.M.S. or sidereal time (N.A. p. II.) from the star's R.A. Under ordinary circumstances it is best to observe stars of the first magnitude only. In practice, then, the problem which would naturally require solution is this. Suppose that on May 15, 1895, we are at sea in about latitude 49° 30′ N. and longitude 9° 36′ W. and it is 8 p.m., we want to know what star of the first magnitude will next pass the meridian. It is first necessary to find the right ascension of the meridian at 8 p.m. Thus:

Ship time	8h. 0m.	R.A.M.S................	3h. 31m. 50·9s.	May 15.
Long. in time .	+ 38m.	Accn. for 8h. 38m. ...	1m. 18·8s.	
			6·2s.	
Greenwich time	8h. 38m.			
		R.A.M.S............	3h. 33m. 16s.	
		Ship time	8h. 0m. 0s.	
		R.A. meridian at 8 p.m.	11h. 33m. 16s.	

Now we turn to the "Nautical Almanac" and look out under the heading "Mean Places of Stars, 1895," pp. 290-4 for the bright star whose right ascension is next greater than the right ascension of the meridian. At p. 292 α Crucis has a right ascension of 12h. 20m. 45s.; by deducting 11h. 53m. 16s. therefrom we find that it passes the meridian 27m. 29s. after 8 p.m. But its declination 62° 31′ 1″·75 S. being south, it is always below the horizon in places to the northward of 27° 29′ N., and it is, therefore, useless to us. The next first magnitude star is α Virginis (Spica), whose right ascension is 13h. 19m. 39·6s., and it, therefore, will pass the meridian at 1h. 46m. 24s. after 8 p.m., that is at 9h. 46m. 24s. p.m. We can readily find the approximate meridian altitude of this star by the latitude and declination. In this case the latitude is about 49° 30′ N., and the declination of Spica is 10° 36′ 47″ S. By adding them together because they are of different names, we obtain the approximate zenith distance, which, deducted from 90°, gives us the true altitude. This is very useful to know, as by putting this angle on the sextant and looking at the southern horizon in this case, because Spica is south of us, we at once see the star we wish to observe in the horizon glass.

It is advisable sometimes to tabulate the stars passing the meridian in this way.

Bright stars that will pass the meridian between 8 p.m. and 12 p.m. on May 15, 1895, in latitude 49° 30′ N., and longitude 9° 36′ W.

Name.	R.A. mer. at 8 p.m.	R.A. star.	Time of mer. pass.	Star's declination.	Approximate altitude and bearing.
	H. M. S.	H. M. S.	H. M. S.		
Spica ...	11 33 16	13 19 40	9 46 24	10° 36′ 48″ S.	29° 53′ 12″ S.
Arcturus	11 33 16	14 10 52	10 37 36	19° 43′ 45″ N.	60° 13′ 45″ S.

Arcturus's declination is north; it is, therefore, subtractive from the latitude of the same name to find the zenith distance. In north latitudes all stars having south declination, and those whose north declination is less than the latitude, are, of course, south of the observer. Those whose north declination is greater than the latitude are north of him.

EXAMPLES.

The observed meridian altitude of Spica on the above date was 30° 1′ 0″ bearing south; I.E.—1′ 10″; height of eye, 14ft.; required the latitude.

Obs. mer. alt.		30° 1′ 0″ S.
I. E.	—	1′ 10″
		29° 59′ 50″
Dip.	—	3′ 41″
		29° 56′ 9″
Refn.	—	1′ 40″
True Alt.		29° 54′ 29″ S.
		90° 0′ 0″
Zen. Dist.		60° 5′ 31″ N.
Dec.		10° 36′ 48″ S.
Lat.		49° 28′ 43″ N.

The observed meridian altitude of Arcturus as above was 60° 16′ 40″.

Obs. mer. alt.		60° 16′ 40″ S.
I. E.	—	1′ 10″
		60° 15′ 30″
Dip.	—	3′ 41″
		60° 11′ 49″
Refn.	—	34″
True Alt.		60° 11′ 15″ S.
		90° 0′ 0″
Zen. Dist.		29° 48′ 45″ N.
Dec.		19° 43′ 45″ N.
Lat.		49° 32′ 30″ N.

CHAPTER XVI.

ON FINDING THE LATITUDE BY OBSERVATION.—BY MERIDIAN ALTITUDE OF A STAR BELOW THE POLE, BY AN ALTITUDE OF POLARIS, AND BY AN EX-MERIDIAN ALTITUDE OF THE SUN.

WE have seen that the altitude of the pole is equal to the latitude of the observer. Now it is obvious that any star whose polar distance is less than the latitude will never sink below the horizon. It is called a circumpolar star, and it will pass the meridian twice in each twenty-four hours, that is, once above and once below the pole; and these meridian passages will take place at intervals of twelve hours. Thus the star α Ursæ Majoris is such a star. Its declination is 62° 19′ 4″ N., and therefore its polar distance is 27° 40′ 56″. In latitude 50° N. the altitude of this star above the pole is 90° − (62° 19′ 4″ N. − 50° N.) = 90° − 12° 19′ 4″ = 77° 40′ 56″; and below the pole it is equal to latitude − polar distance, that is, 50° − 27° 40′ 56″ = 22° 19′ 4″. The time of the meridian passage above the pole is calculated in the ordinary way; but the meridian passage below the pole occurs twelve hours after or before.

In Fig. 31, constructed on the plane of the horizon, let

X and Y be the position of a circumpolar star on the meridian above and below the pole. The latitude is ascertained when the meridian altitude Y N above the pole is taken in the ordinary way. But when the meridian altitude X N below the pole is taken, the polar distance

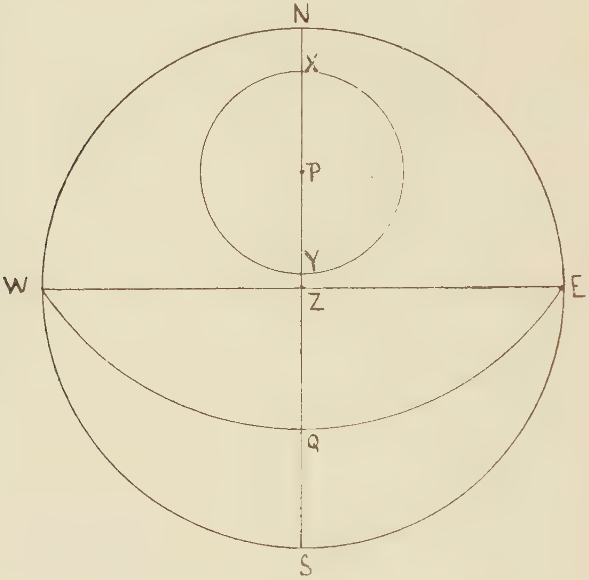

Fig. 31.

P X is added to the corrected altitude X N and the latitude P N is obtained.

Thus the right ascension of α Ursæ Majoris is 10h. 57m. 14s., and on the evening of May 15, 1895, the R.A. meridian at 8 p.m. is, as we have seen in the last chapter, 11h. 33m. 16s. This star then is on the meridian above the pole 36m. 2s. before 8 p.m., that is, at 7h. 23m. 58s.

p.m., and below the pole about twelve hours afterwards at 7h. 25m. 58s. a.m. on May 16.

EXAMPLES.

(1) On May 15th, 1895, at about 9 p.m. in latitude by account 50° 0′ N., the observed meridian altitude of α Cassiopeiæ under the North Pole was 16° 14′ 20″, I.E. – 1′ 10″, height of eye, 14ft.; required the latitude?

To find time of meridian passages below the North Pole:

Right Ascension α Cassiopeiæ	0h. 34m. 32·8s.
Right Ascension Mean Sun	3h. 31m. 50·9s.
Time of Meridian Passage............ (above the Pole on May 14th)	21h. 2m. 41·9s.

The star will pass the meridian under the pole 12h. 2m. after the time of its meridian passage above it. The two minutes is allowed for the acceleration due to the earth's movement in her orbit, which causes all the stars to pass the meridian about four minutes later on each successive day. In any year, therefore, a fixed star will be on the meridian of any place at almost the same time as it was on the same date in the previous year.

It is quite accurate enough to take out the R.A.M.S. or sidereal time from the "Nautical Almanac" without correcting it for acceleration, as we only want to ascertain the time of the star's meridian passage approximately. The sextant must tell us when the star is on the meridian, that is, in the case of a star below the pole, when it attains its least altitude. It must be remembered also that stars cross the meridian below the pole from west to east.

To ascertain the approximate altitude:

In Fig. 31, X N = P N − P X
Altitude = Latitude − Polar distance.
Polar distance = 90° − declination.

Declination α Cassiopeiæ =	55° 57′ 40″ N.*
	90° 0′ 0″
Polar distance ,,	34° 2′ 20″
Latitude by account	50° 0′ 0″
Approximate altitude	15° 57′ 40″

We place this altitude on the sextant and sweep the horizon under Polaris, when we shall find the star required, reflected in the horizon glass, this reflection we must keep touching the horizon till the star has ceased to fall, and the arc on the sextant is the altitude required.

Latitude = Altitude + Polar Distance.

Obs. mer. alt. α Cassiopeiæ below pole	16° 14′ 20″
I.E.	1′ 10″
	16° 13′ 10″
Dip	3′ 41″
Apparent altitude	16° 9′ 29″
Refn.	3′ 18″
True altitude......	16° 6′ 11″
Polar distance ...	34° 2′ 20″
Latitude	50° 8′ 31″ N.

(2) On November 9th, 1895, at 9.10 p.m., in latitude by account 54° 0′ S. the observed meridian altitude of α Crucis below the South Pole was 26° 43′ 20″, I.E. + 1′ 40″; height of eye, 17ft.; required the latitude.

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* "Nautical Almanac," p. 290.

		Obs. mer. alt.	26° 43′ 20″
Declination α Crucis	62° 31′ 2″ S.*	I.E.	+ 1′ 40″
	90° 0′ 0″		26° 45′ 0″
Polar distance	27° 28′ 58″	Dip.............	4′ 4″
		Appt. alt.......	26° 40′ 56″
		Refn...	1′ 55″
		True alt.........	26° 39′ 1″
		Polar distance	27° 28′ 58″
		Latitude	54° 7′ 59″ S.

By Altitude of the Pole Star (Polaris).

Polaris is not exactly at the celestial pole, otherwise the corrected altitude of this star would be the latitude of the place. It is situated, however, within about 1° 15′ of the true pole, and tables have been calculated by means of which the latitude of an observer can be determined by an altitude of this star at any time when it is visible. On pages 483, 484, and 485 of the "Nautical Almanac" for 1895, these corrections are to be found. They depend on the sidereal time, that is, the R.A. meridian at the time of observation, on the altitude, and on the period of the year. One minute of arc (1′) is always subtracted from the observed altitude in order that the two last corrections shall be always additive.

Example.

On May 20, in longitude 17° 20′ W., at about 10.20 p.m., the observed altitude of Polaris was 49° 47′ 10″, I.E. + 2′ 40″, height of the eye 17ft.

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* "Nautical Almanac," p. 292.

Ship time	10h. 20m.	Obs. alt.		49° 47′ 10″
R.A.M.S.	3h. 52m.	I.E..................	+	2′ 40″
R.A.Mer.	14h. 12m.			49° 49′ 50″
		Dip..................	−	4′ 4″
				49° 45′ 46″
		Refn.	−	49″
				49° 44′ 57″
		Const. Corrn.......	−	1′ 0″
				49° 43′ 57″
R.A.Mer. or sidereal time, 14h. 12m., 1st Cor.			+	1° 13′ 0″
				50° 56′ 57″
Sidereal time, 14h. 12m., altitude, 51°, 2nd Cor.......			+	2″
Sidereal time, 14h. 12m., date, June 1, 3rd Cor.......			+	1′ 10″
		Latitude.................		50° 58′ 9″ N.

Polaris is extremely useful to navigators. In clear weather and with a good horizon the latitude can be ascertained at any time by observing the altitude of this star. Only an approximation of the sidereal time is required, and the sidereal time at Greenwich noon, as taken from the "Nautical Almanac" need not be corrected for acceleration.

BY AN EX-MERIDIAN ALTITUDE OF THE SUN.

This is an essential problem, as it is impossible to obtain for certain the altitude of the sun when that body is exactly on the meridian except when the sky is cloudless. But even in cloudy weather it is very often practicable to take its altitudes within a short time of apparent noon. The shorter the time the better. As a general rule, *the minutes of time from apparent noon should not exceed the number of degrees in the meridian zenith distance,* the meridian zenith distance being the angular distance from the zenith to the centre of the sun or any other heavenly body when on the meridian. It is the sum of the latitude and declination when they are of different names, and their difference when of the same name. In other words, the lower the meridian altitude the longer interval of time between apparent noon and the time of observation may be permitted without materially affecting the accuracy of the result. For example, in latitude 50° N., declination being 20° N., the meridian zenith distance is 30°, and an observation taken within 30 minutes of apparent noon is quite reliable.

The elements on which this problem depends are the latitude by account, the declination of the sun, the altitude of that body, and the time accurately from apparent noon. The latitude we get by the dead reckoning, the declination from the "Nautical Almanac," the altitude by observation, and the time from noon by the chronometer or by a watch whose error on apparent time is known.

I think that perhaps the simplest rule to work with is that given by Norie:

RULE.

1. Find Greenwich time.

2. Correct declination of sun and equation of time for Greenwich time.

3. Correct observed altitude of sun and find zenith distance.

4. Find apparent time at which the observation was taken, and hence the time from apparent noon.

(*a*) If chronometer time is used, find ship apparent time (S.A.T.) by applying error of chronometer on Greenwich mean time (G.M.T.), longitude in time and equation of time.

(*b*) If a watch whose error on apparent time is known, correct the time by watch for such error, and if the ship's longitude has changed since the error was ascertained, the difference of longitude in time must be applied; it is to be added if the run is to the eastward, and subtracted if to the westward.

5. Add together constant log. ·301030, log. cos. of the latitude, log. cos. of the declination, log. cosec. of the sum of latitude and declination if of different names, or of their difference if of the same name, and log. haversine of the time from noon (log. horary angle, Norie). The sum of these logs., rejecting tens, is the log. sine of an angle which is to be added to the corrected altitude to find the true meridian altitude.

6. Deduct the true altitude thus found from 90°, which gives us the meridian zenith distance, and apply the declination as in a meridian altitude, the result being the latitude *at the time of observation.*

EXAMPLE.

1. On Dec. 15, 1895, in latitude by account 46° 50′ N. and longitude 34° 15′ W., when a chronometer, whose error on G.M.T. was 2h. 17m. 20s. fast, showed 3h. 57m. 55s., the obs. alt. of the sun's L.L. was 19° 34′ 30″, I.E. + 3′ 10″, height of eye 15ft.

Chron. showed	3h. 57m. 55s.
Error of Chron.	2h. 17m. 20s. fast.
Dec. 15, G.M.T.	1h. 40m. 35s.
Long. in Time (West)	2h. 17m. 0s.
December 14, S.M.T.	23h. 23m. 35s.
Corrected Eq. Time	+ 4m. 45s.
S.A.T.	23h. 28m. 20s.
	24h. 0m. 0s.
Time from app. noon	0h. 31m. 40s.

Declination.

December 15		23° 17′ 9″ S.
Var. for 1h. 40m.		13″
At sights		23° 17′ 22″ S.

Equation of Time.

December 15	+	4m. 42s.·88
Var. for 1h. 40m.		2s.·06
At sights	+	4m. 44s.·94

Const. log.			·301030
Declination	23° 17′ 22″ S.	Cosine	9·963088
Latitude	46° 50′ 0″ N.	Cosine	9·835134
	70° 7′ 22″	Cosecant	10·026677
Time from app. noon.	31m. 40s.	Log. hav. (Inman) Log. horary angle (Norie).	7·678088
Log. sine			7·804017
Reduction or angle to be added to cor. alt.			0° 21′ 53″

Obs. alt.		19° 34′ 30″
I.E.	+	3′ 10″
		19° 37′ 40″
Cor.		+ 9′ 48″
Tr. alt.		19° 47′ 28″
Reduction		21′ 53″
Mer. alt.		20° 9′ 21″ S.
		90° 0′ 0″
Zen. dist.		69° 50′ 39″ N.
Declination		23° 17′ 22″ S.
Latitude at time of observation 11h. 28m. 20s. a.m., Dec. 14		46° 33′ 17″ N.

With a latitude no less than 17′ in error we obtain an almost perfectly accurate result, for if my readers will work this problem with the latitude found above, viz., 46° 33′ 18″, the reduction thus obtained only varies 9″ from that given by the erroneous latitude.

2. On April 20th, 1895, in latitude by account 29° 19′ S. and longitude 57° 20′ E., when a watch showed 11h. 15m. 40s., whose error on apparent time was found to be 24m. 5s. slow, the ship having run east 24′ since, the observed altitude of the sun's L.L. was 49° 3′ 20″, I.E. – 2′ 10″; height of eye, 21ft.

Watch showed	11h. 15m. 40s.
Error of watch	24m. 5s. slow.
	11h. 39m. 45s.
Run	1m. 52s.*
April 19, S.A.T.	23h. 41m. 37s.
Long. in time (East)	3h. 49m. 20s.
April 19, G.A.T.	19h. 52m. 17s.

Declination.

April 19	11° 10′ 59″·1
Variation in 19·9h.	17′ 12″·2
At sights	11° 28′ 11″ N.

S.A.T.	23h. 41m. 37s.
	24h. 0m. 0s.
Time fr. app. noon	0h. 18m. 23s.

Const. log		·301030
Declination	11° 28′ 11″ N. Cos.	9·991240
Latitude by account	29° 19′ 0″ S. Cos.	9·940480
Sum	40° 47′ 11″ Cosec.	10·184926
Time fr. noon	0h. 18m. 23s. H.A.†	7·206190
Log. sine reduction		7·623866
Reduction		0° 14′ 28″

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* Due to 28′ difference of longitude.

† Haversine (Inman).

Obs. Alt.	49°	3′	20″
I.E.	—	2′	10″
	49°	1′	10″
Corrn., 21ft.	+	10′	42″
Tr. alt.	49°	11′	52″
Reduction	0°	14′	28″
Mer. alt.	49°	26′	20″ N.
	90°	0′	0″
Zen. Dist.	40°	33′	40″ S.
Declination	11°	28′	11″ N.
	29°	5′	29″ S.

This latitude is correct within 6″. The correction in time for the run of the ship is the difference of longitude in time due to the departure of 24′ in latitude 29° S.; and it is added because the run is to the eastward.

CHAPTER XVII.

On Finding the Latitude by Meridian Altitude of the Moon or of a Planet.

The moon is so near to the earth, and moves so rapidly in her orbit that great care and precision are required in correcting the declination and right ascension of that body for the time of observation. In Fig. 26, p. 134, if we suppose X to be the position of the centre of the moon, it is obvious that her parallax, the angle A X O, owing to her short distance from the earth, is very large. In the Nautical Almanac, at p. III. in each month we find the moon's horizontal parallax, that is the angle subtended at the moon, when that body is on the horizon by the semi-diameter of the earth, A O in Fig. 26. A moment's consideration of this figure will show us that this angle gets less as the moon's altitude increases till when she is at the zenith, the point in the celestial concave directly over A, it entirely disappears.

Again, the moon is nearer to a spectator on the surface of the earth when she is at his zenith than when on his horizon by about 4000 miles, the semi-diameter of the earth, and as the moon's semi-diameter given in the Nautical Almanac at p. III. in each month is the angular measurement of such semi-diameter from the earth's centre, a correction called the augmentation of the moon's semi-diameter due to her angular distance from the horizon, or altitude, must be added to that taken from the Nautical Almanac.

In all other respects the latitude is found by a meridian altitude of the moon in precisely the same manner as by that of the sun or of a star.

The time of the moon's meridian passage is given in the Nautical Almanac, in the last two columns of p. IV.

EXAMPLE.

(1) On June 1st, 1895, in longitude 55° 10′ W., the observed meridian altitude of the moon's lower limb was 45° 10′ 30″ bearing S., index error + 1′ 30″, height of eye 17ft. Required the latitude?

(*a*) To find the time of the moon's meridian passage and hence the Greenwich time for correction of declination, &c.

At p. IV. in the Nautical Almanac we take out the time of two successive upper meridian passages, and take their difference which is the daily variation.

We next enter Table 23, Inman; Table XVI., Norie; with the daily variation at the top and the longitude at the side, and take out the correction to be added to the first meridian passage if the longitude is west, or subtracted if east.

MOON'S MERIDIAN PASSAGE.

On June 1st	7h.	17m. ·5
,, 2nd	8h.	4m. ·4
Daily variation	0h.	46m. ·9
On June 1st	7h.	17m. ·5
Correction for 46m. ·9 daily variation and 55° W. longitude in the above tables	+	7m. ·0
Time of Meridian Passage in longitude 55° W.	7h.	24m. ·5
Longitude 55° 10′ W. in time	3h.	40m. ·6
Greenwich time	11h.	5m. ·1

(*b*) Our next operation is to correct the declination for the Greenwich time thus found.

In the Nautical Almanac in each month from p. V. to p. XII. are tabulated the right ascension and declination of the moon for every hour, and the variation is given for each ten minutes. To correct the moon's declination for Greenwich time.

On p. V., June 1st, at 11 hours moon's declination is 2° 8′ 53″ ·9 S.

The variation in 10 minutes is 165″ ·9
5m. = 10m. × ·5 ·5

6,0)8,2·95

∴ The variation in 5 minutes is 1′ 22″ ·95
To be added because the declination is increasing.

Moon's declination on June 1st at 11 hours			2° 8′ 53″ ·9 S
„	„	variation in 5 minutes	1′ 22″ ·9
„	„	at 11h. 5m.	2° 10′ 16″ ·8 S

We multiply the variation for 10 minutes taken from the Nautical Almanac by the number of minutes in the Greenwich time, and divide the product by 10 by moving the decimal point in the product one figure to the left.

(*c*) To correct semi-diameter and horizontal parallax for Greenwich time.

At p. III. Nautical Almanac :

	Semi-diameter.	Horizontal Parallax.
On June 1st, at noon	16′ 6″ ·1	58′ 59″ ·6
Midnight	16′ 3″ ·9	58′ 51″ ·3
Difference in 12 hours	2″ ·2	8″ ·3

Now 11h. 5m. is one hour from midnight nearly, we therefore have to subtract $\frac{11}{12}$ of the differences from

the semi-diameter and horizontal parallax at noon respectively, because they are decreasing. Therefore:

	Semi-diameter.
At noon on 1st June	16′ 6″·1
1½ of 2″·2	— 2″
At time of observation	16′ 4″
Augmentation due to 16′ semi-diameter and 45° altitude *	11″·8
Corrected semidiameter	16′ 15″·8

	Horizontal Parallax.
At noon on 1st June	58′ 59″·6
1½ of 8″·3	— 7″·6
Corrected Horizontal Parallax	58′ 52″

In Table 22, Inman, Table D, Norie, enter with the moon's semi-diameter and the approximate altitude, for the augmentation to be added to the semi-diameter as above.

(*d*) To correct the moon's altitude.

Observed Mer. Alt. Moon's L.L. =	45° 10′ 30″ S
I.E.	+ 1′ 30″
	45° 12′ 0″
Dip. 17ft.	— 4′ 4″
	45° 7′ 56″
Semi-diameter †	+ 16′ 16″
Apparent Altitude	45° 24′ 12″
Correction in Altitude for horizontal parallax 58′ 52″	39′ 47″ 37″
True Altitude	46° 4′ 36″ S
	90 0 0
Zenith Distance	43° 55′ 24″ N
Declinatton	2° 10′ 17″ S
Latitude	41° 45′ 7″ N

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* Inman, Table 22; Norie, Table D.

† To be added because the lower limb is observed.

The above correction in altitude is found in Table 24 Inman, at p. 87, entered with 45° 24′ apparent altitude at the side, and 58′ horizontal parallax at the top, which gives us 39′ 47″; then opposite the 52″ of horizontal parallax in the right hand column we find 37″.

In Table XXX., Norie, the correction is taken out in the following manner:

At p. 254—

45° 20′ apparent altitude at the side 58′ horizontal parallax at the top } gives	39′ 40″
In the next column 52″ of parallax gives	36″
And in the right-hand column 4′ of altitude gives ...	8″
Total correction	40′ 24″

which gives the same result as Inman.

This correction in altitude is the angle AXO, to which I referred at the beginning of this chapter.

(2) On November 7th, 1895, in longitude 89° 40′ E. the observed meridian altitude moon's upper limb was 51° 14′ 20″ bearing north, index error − 0′ 40″, height of eye 12ft. Required the latitude?

Mer. Passage, 7th	16h. 33m.	(N. A., 185)
,, 8th	17h. 28m.	,,
Daily variation........................	0h. 55m.	

Mer. Passage, 7th	16h. 33m.	
Corrn. 55m. daily variation and 89° 40′ E. longitude }	− 14m.	(Inman, p. 66) (Norie, p. 80)
Time of meridian passage at observer...................... }	16h. 19m.	
Longitude in time	5h. 58m. ·7	
Greenwich time	10h. 20m. ·3	

Declination.

Nov. 7th at 10 hours	26° 0′ 30″ ·8 N.	(N. A. p. 187)
Variation in 20 minutes	2′ 15″ ·6	
At sights	25″ 58′ 15″ ·2 N.	
Variation in 10 minutes	67″· 78	
	20	
,, 20 minutes.........	135″ ·560	
,, ...	= 2′ 15″ ·6	

Semi-diameter.

On Nov. 7th, at noon	15′ 46″ ·8	(N. A., p. 184)
,, midnight	15′ 50″ ·7	,,
Difference for 12 hours	0′ 3″ ·9	
At noon	15′ 46″ ·8	
Difference for 10h. ·3	+ 3·3	
	15′ 50″ ·1	
Augmentation, altitude 51°, semi-diameter 15′ 50″	12″ ·6	(Inman, p. 66) (Norie, p. 2*)
Corrected semi-diameter..............	16′ 2″ ·7	

Horizontal Parallax.

On Nov. 7th, at noon	57′ 48″ ·9	(N. A., p. 184)
,, midnight	58′ 3″ ·2	,,
Difference for 12 hours	0′ 14″ ·3	
At noon	57′ 48″ ·9	
Difference for 10h. ·3	12″ ·3	
Corrected horizontal parallax	58′ 1″ ·2	

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* To be subtracted because the Upper Limb is observed.

Obs. Alt. Moon's Upper Limb.	51° 14′ 20″ N.	
I. E.	— 0′ 40″	
	51° 13′ 40″	
Dip (12 feet)	— 3′ 25″	
	51° 10′ 15″	
Semi-diameter*	— 16′ 3″	
Apparent Altitude	50° 54′ 12″	
Correction in Altitude..	35′ 48″	(Inman, p. 90)
	1″	(Norie, p. 256)
True Altitude	51° 30′ 1″ N.	
	90° 0′—0″	
Zenith Distance	38° 29′ 59″ S.	
Declination	25° 58′ 15″ N.	
Latitude	12° 31′ 44″ S.	

BY A PLANET.

Some of the planets being comparatively near to the earth their parallax has to be taken into consideration when correcting any altitudes of them which have been observed. In the Nautical Almanac, 1895, pp. 270 to 289, all the information is to be found which is requisite to work out problems in navigation involving altitudes of planets. The time of their meridian passage at any place is ascertained in precisely the same manner as that of any other heavenly body : that is to say, that when such a body is on the celestial meridian of any place, its right ascension is equal to the sidereal time of the place or right ascension of the meridian, which is also equal to the right ascension of the mean sun plus mean time at the place (R.A.M.S. + S.M.T.) : and, therefore, S.M.T. = R.A. heavenly body − R.A.M.S.

1. On January 18th, 1895, p.m., in longitude 154° 25′ W., the observed meridian altitude of Jupiter's Lower Limb

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* To be substracted because the Upper Limb is observed.

was 71° 15′ 20″ bearing south, Index Error + 1′ 40″, height of eye 25ft. Required the latitude ?

(*a*) To find the time of meridian passage nearly, and Greenwich time.

Right Ascension Jupiter January 18th	5h. 51m.	(N.A., p. 280)
R.A.M.S.	19h. 51m.	(N.A., p. 3)
Ship Mean Time (S.M.T.) of Meridian Passage	10h. 1m.	on Jan. 18th
Longitude in Time West	10h. 18m.	
Mean Time Greenwich (G.M.T.)	20h. 19m.	on Jan. 18th

(Neglecting the seconds of time).

(*b*) To correct declination for Greenwich time :

Jupiter declination, June 18th	23° 16′ 43″ ·5 N	(N.A., p. 280)
Variation in one hour. 0″·2 × 20·3 hours (correction)	4″·1	
At sights	23° 16′ 48″ N.	

0″·2 is the hourly change of declination (page 280, N.A.)
20·3 is the time in hours at Greenwich.

(*c*) To correct the altitude and find the latitude :

Obs. Alt. Jupiter's L.L.	= 71° 15′ 20″ S.
I.E.	+ 1′ 40″
	71° 17′ 0″
Dip. 25ft.	− 4′ 55″
	71″ 12′ 5″
Semi-diameter (N.A., p. 280)	+ 22″
Apparent Altitude	71° 12′ 27″
Correction due to Parallax...	1″ *
True Altitude	71° 12′ 28″ S
	90° 0′ 0″
Zenith Distance	18° 47′ 32″ N
Declination	23° 16′ 48″ N
Latitude	42° 4′ 20″ N

Corr. in Altitude for Horizontal Parallax 2″·1, Alt. 71°	Inman, Table 20. Norie, Table XLVIII.

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* Horizontal Parallax, N.A. p. 280, is 2″·1.

In the above examples the horizontal parallax and semi-diameter are taken out at p. 280 in the Nautical Almanac (N.A. p. 280) and the correction in altitude taken found in Table 20 Inman, or Table XLVIII. Norie.

(2) On February 16th, 1895, a.m., in longitude 45° W., the observed meridian altitude of Saturn's Lower Limb was 35° 21′ 50″, bearing north, Index Error — 1′ 20″, height of eye 17ft. Required the latitude ?

R.A. Saturn. Feb. 16th	14h. 23m.	(N.A. p. 283)
R.A.M.S. „	21h. 45m.	(N.A. p. 21)
S.M.T. „ 15th	16h. 38m.	
Longitude in Time...	3h. 0m.	
G.M.T. =	19h. 38m.	

Obs. Mer. Alt. Saturn =	35° 21′ 50″ N	
Index Error	— 1′ 20″	
	35° 20′ 30″	
Dip........................	— 4′ 4″	
	35° 16′ 26″	
Semi-diameter	8″	(N.A. p. 283)
Apparent Altitude ...	35° 16′ 34″	
Corrn. in Altitude ...	0 0	Hor. Par. 0″·9 N.A. p. 283
True Altitude............	35° 16′ 34″ N	
	90 0 0	
Zenith Distance.........	54° 43′ 26″ S	
Declination...............	11° 31′ 3″ S	
Latitude	66° 14′ 29″ S	

In this example the change of declination, &c., is very small, so that the data can be taken out of the Nautical Almanac by inspection. The distance of Saturn is so enormous that the horizontal parallax should be disregarded.

Altitudes of the moon and planets are sometimes of great value for the purposes of navigation, and the mariner ought to accustom himself to determine his position by observations of these bodies. When the horizon is clear and distinct, especially about the time of sunrise and sunset, they should be quite accurate.

In the above examples the semi-diameter and parallax of the planets might be safely neglected, and, except in the case of Mars and Venus, when they are nearest to the earth, this is true in working out all planet observations, except when very great precision is required.

CHAPTER XVIII.

ON FINDING THE LONGITUDE BY CHRONOMETER OBSERVATION.

IN Chapters XIII. and XIV., I have explained the relationship between time and longitude, and that if we knew the mean time at any two places on the earth's surface at the same moment, we also knew the difference between their respective longitudes. Now the longitude of any place is the arc of the equator intercepted between the prime meridian and the meridian of such place (see Chapter VII.). Knowing the mean time at Greenwich, and, at the same moment, the mean time at any other place, we are able at once to find the difference of longitude between Greenwich and that place, and this difference is the longitude of the place, because the meridian of Greenwich is our prime meridian. Some foreign countries have other prime meridians, nevertheless that of Greenwich is almost universally adopted.

To determine the longitude of any place, then, we must know the mean time at Greenwich, and, at the same moment, the mean time at the ship, or G.M.T. and S.M.T.

TO FIND GREENWICH MEAN TIME.

The Greenwich mean time when certain astronomical phenomena occur is given in the Nautical Almanac. Such are, among others, the angular distances of the moon from the sun, or from certain bright stars; these are called lunar distances: the eclipses of Jupiter's satellites; and moon culminating stars. The first of these, that is determining the Greenwich mean time by the observations of lunar distances, was universally used before the introduction of the chronometer; but very great accuracy of observation is required, which the constant movement of a ship render very difficult of attainment, and consequently the results derived therefrom were at best only approximate. I do not propose, therefore, to touch upon this subject, nor are the two other methods mentioned above of much practical use at sea.

Every seagoing ship should carry with her a wonderfully delicate timekeeper—in other words, a chronometer, whose error on G.M.T. is known, and whose daily rate has been ascertained, so that G.M.T. can be noted at any moment on board the vessel. A good chronometer will maintain its daily rate with marvellous precision through all the vicissitudes of temperature to which it may be exposed, nor do the pitching and rolling motions of a vessel have any effect upon its accuracy. It is a miracle of human ingenuity and skill.

TO FIND SHIP MEAN TIME.

1. By an altitude of the sun.

The polar angle formed by the meridian of the place, and that meridian passing through the centre of the apparent or real sun, measured westward from the former, is the sun's hour angle or ship apparent time (S.A.T.). In Fig. 29, p. 146, let X^2 be the sun west of the meridian. The angle ZPX_2 is the apparent time of an observer whose zenith is Z. To find this, we have in the spherical triangle PZX_2, PZ the co-latitude, PX_2 the polar distance, and ZX_2 the zenith distance, the three sides. The formulæ employed to solve this triangle have been proved in Chapter VIII., and are:

(1) Using haversines—

$$\begin{array}{l}\text{Log. Hav. Hour} \\ \quad\text{Angle} \ldots\ldots\ldots\end{array} \left\{ \begin{array}{l} = \text{Log. sec. declination} + \text{log. sec. latitude} \\ + \tfrac{1}{2}\text{ log. Hav.} \left\{ \begin{array}{l} \text{Zenith Distance} + (\text{lati-} \\ \quad \text{tude} \pm \text{declination}) \ldots\ldots \end{array} \right\} \\ + \tfrac{1}{2}\text{ log. Hav.} \left\{ \begin{array}{l} \text{Zenith Distance} \sim (\text{lati-} \\ \quad \text{tude} \pm \text{declination}) \end{array} \right\} - 20 \end{array} \right.$$

(2) Not using haversines—

$$\begin{array}{l}\text{Log. Horary} \\ \text{Angle (Norie)} \ldots = \end{array} \left\{ \begin{array}{l} \text{Log. sec. latitude} + \text{log. cosec. polar dis-} \\ \quad \text{tance} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ + \text{log. cos. } \tfrac{1}{2}\ (\text{altitude} + \text{latitude} + \text{polar} \\ \quad \text{distance}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ + \text{log. sin.} \left\{ \begin{array}{l} \tfrac{1}{2}\ (\text{altitude} + \text{latitude} + \text{polar} \\ \quad \text{distance}) - \text{altitude} \end{array} \right\} - 30 \end{array} \right.$$

We will now proceed to the practical use of these formulæ. The former is almost exclusively used in the Royal Navy, and is, in my opinion, less liable to error in working than the latter, which is generally adopted in the mercantile marine.

The best time for taking an observation of any heavenly body for the purpose of determining its hour angle is when it bears east or west true, because an error in the altitude causes then the smallest relative error in the hour angle, provided, however, that the altitude at that time is not less than 10°. The refraction caused by the atmosphere varies considerably, and is liable to cause grave error when a heavenly body is too near the horizon. As a matter of custom, sights are taken at about 8 a.m., if the sun has attained a suitable altitude.

RULE FOR SUN CHRONOMETER (USING HAVERSINES).

1. Find Greenwich mean time from chronometer time given.

2. Correct declination and equation of time for G.M.T.

3. Find the true altitude of the sun by correcting the observed altitude for index error and correction in altitude; deduct it from 90°, and so find zenith distance.

4. Add together log secant latitude, log secant declination.

$$\tfrac{1}{2}\text{ log. haversine}\left(\left.\begin{matrix}\text{latitude} + \text{declination}\\ \text{latitude} \sim \text{declination}\end{matrix}\right\}^{*} + \text{zenith distance}\right)\text{ and}$$

$$\tfrac{1}{2}\text{ log. haversine}\left(\left.\begin{matrix}\text{latitude} + \text{declination}\\ \text{latitude} \sim \text{declination}\end{matrix}\right\}^{*} \sim \text{zenith distance}\right).$$

The sum of these logs less the tens in the characteristic is the log. haversine of the sun's hour angle or ship's apparent time; it must be taken out *from the bottom of the table* if the observed body is *east of the meridian;* and *from the top of the table* if it is *west of the meridian.*

5. To S.A.T. apply the corrected equation of time which gives us S.M.T., and the difference between this and G.M.T. is the longitude in time, which converted into arc is the longitude to be marked West if Greenwich time is the greater, and East if otherwise.

EXAMPLE 1.

On August 6th, 1895, at about 8 a.m., in lat. 49° 44′ N., and longitude by account 16° 44′ W., the obs. alt. of the p.c

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* The latitude and declination are to be *added together* if they are of *different names;* and *their difference* is to be taken if of the *same name.*

sun's L.L. was 31° 1', I.E. – 1' 40", height of the eye 18ft., when a chronometer showed 8h. 28m. 37s. whose error on Greenwich mean time was 39m. 12s. slow. Required the longitude?

1.

Chron. showed	8h.	28m.	37s.
Error of chron.		+39m.	12s.
G.M.T..................	21h.	7m.	49s. on August 5th, 1895.

The time at ship being a.m. the Greenwich time must also be a.m., as the difference of longitude is only about an hour. We, therefore, add 12 hours to the time as shown by the chronometer, and so obtain the Greenwich mean time of August 5th. From the Nautical Almanac, p. 129, the declination and equation of time are taken out for August 5th, and the hourly variation (p. 128) is multiplied by 21·1 the number of hours and decimals of an hour after Greenwich mean noon when the observation was taken.

2.

			m.	s.
Declination.........	16° 59' 19"·6 N.	Equation of time...	5	48·84+
Varn. 40"·6 × 21·1.=	– 14' 16"·7	·249s × 21·1		–5·25
At sights	=16° 45' 03" N.	At sights	=5	43·59+

(marked + because it is to be added to apparent time: see Nautical Almanac, p. 129).

3.

Obs. alt. $\underline{\odot}$	= 31°	1'	0"	
I. E.	=	– 1'	40"	
	30°	59'	20"	
Correction 18ft. ...	=	+10'	19"	(Inman, Table 17)
True alt.	= 31°	9'	29"	
	90°	0'	0"	
Z.D.	= 58°	50'	30"	

4.

Lat.	49°	44′ N.		sec.	·189535	(Inman, Table 32)
Dec.	16°	45′ N.		sec.	·018829	(„ „ „)
	32°	59′				
Z.D.	58°	50′	30″			
	91°	49′	30″	½ hav.	4·856291	(„ „ 33)
	25°	51′	30″	½ hav.	4·349756	(„ „ „)
				Log hav. S.A.T.	9·414411	(„ „ 34)

5.

S.A.T.	= 19h.	54m.	55s.	on August 5th
E.T.	=	+ 5m.	44s.	„ „ „
S.M.T.	= 20h.	0m.	39s.	„ „ „
G.M.T.	= 21h.	7m.	49s.	„ „ „
Long. in Time	= 1h.	7m.	10s.	
∴ Longitude........	= 16°	47′	30″ W.	

RULE FOR SUN CHRONOMETER (WITHOUT USING HAVERSINES).

1, 2, 3 are the same as in the previous rule.

4. Add together the log secant latitude, log cosecant polar distance, log cosine of half the sum of the true altitude, latitude and polar distance, and the log sine of the difference between the true altitude and the half sum of the true altitude, latitude and polar distance; the sum of these logs rejecting the tens in the characteristic is the log horary angle (Norie) of the S.A.T., which must be taken out from the top of the table if the time is p.m., and from the bottom if a.m.

Then proceed as in previous rule.

EXAMPLE.

We will work out the example given above by this method :

Chronometer showed		8h.	28m.	37s.	
Error of chronometer	+		39m.	12s.	
G.M.T.		21h.	7m.	49s.	on August 5th, 1895
Declination as above		16°	45′	3″	N.
		90°	0′	0″	
Polar distance		73°	14′	57″	
Equation of time as above...	+		5m.	43·6s.	

Obs. alt. ⊙...........		31°	1′	0″				
I.E.			1′	40″				
		30°	59′	20″				
Corrn 18ft............	+		10′	91″	(Norie, Table IX.)		parts for seconds.	
Tr. alt.		31°	9′	29″				
Latitude..............		49°	44′	0″	Secant ...	·189535		*
Polar distance		73°	14′	57″	Cosecant..	·018848...	17	*
Sum		154°	8′	26″				
½ sum..................		77°	4′	13″	Cosine ...	9·349893...	120	*
½ sum ~ altitude ...		45°	54′	44″	Sine	9·856262		*
						29		
						9·414567...	137	
						137		
					Log horary angle=	9·414430		†

S.A.T.	= 19h.	54m.	55s.	on August 5th	
E.T.	= +	5m.	44s.		
S.M.T.	= 20h.	0m.	39s.	,, ,, ,,	
G.M.T.	= 21h.	7m.	49s.	,, ,, ,,	
Long. in time	= 1h.	7m.	10s.		
Longitude	= 16°	47′	30″	W.	

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* Norie, Table XXV. † Norie, Table XXXI.

In a star chronometer the same rule is followed, and the star's hour angle is found, whence S.M.T. is ascertained from the formula H.A. star + R.A. star = R.A.M.S. + S.M.T.

Example 2.

On August 6, 1895, at about 3 a.m., in latitude 49° 30′ N. and longitude by account 16° 12′ W., when a chronometer showed 3h. 27m. 17s., whose error on G.M.T. was 38m. 50s. slow, the observed altitude of Aldebaran (east of meridian) was 27° 1′ 20″ I.E. – 1′ 40″, height of the eye 18ft. Required the longitude.

Chronometer showed ………………	3h. 27m. 17s.	
Error of chronometer …………	+ 38m. 50s.	
G.M.T. …………………………… =	16h. 6m. 7s.	on August 5.
Right ascension Aldebaran, Nautical Almanac, p. 325 …	4h. 29m. 56s.	
Declination ditto …………………	16° 18′ 7″ N.	

R.A.M.S. August 5 =	8h. 55m. 8·66s.	(Nautical Almanac), p. 129
Accn. for 16h. …… =	2m. 37·70s.	(Norie Table, XXXVIII.)
,, ,, 6m. …… =	·99s.	,, ,, ,,
,, ,, 7s. …… =	·02s.	,, ,, ,,
At sights ……… =	8h. 57m. 47·37s.	

Obs. alt. …………… =	27° 1′ 20″	
I.E. ……………… –	1′ 40″	
	26° 59′ 40″	
Dip. ……………… –	4′ 11″	(Norie Table, VIII.)
	26° 55′ 29″	
Refn. ……………… –	1′ 52″	(,, ,, XVIII.)
Tr. alt. ……………	26° 53′ 37″	
	90° 0′ 0″	
Zen. dist. …………	63° 6′ 23″	

Lat.	49° 30′ 0″ N.	Secant	= ·187456
Decn.	16° 18′ 7″ N.	Secant	= ·017821
	33° 11′ 53″		
Zen. dist.	63° 6′ 23″		
	96° 18′ 16″	½ hav..................	= 4·872109
	29° 54′ 30″	½ hav..................	= 4·411698

Log. Hav. Aldebaran's H.A. (east of meridian) 9.489084 ✓

Aldebaran's H.A....	=	19h. 30m. 8s.
R.A.	=	4h. 29m. 56s.
R.A. meridian	=	24h. 0m. 4s.
R.A.M.S.	∸	8h. 57m. 47s.
S.M.T.	=	15h. 2m. 7s. on August 5.
G.M.T.	=	16h. 6m. 7s. „ „ „
Long. in time......	∸	1h. 3m. 50s.
Longitude	=	15° 57′ 30″ W.

The last example, worked without haversines:

Aldebaran's declination	16° 18′ 7″ N.
	90° 0′ 0″
„ polar distance	73° 41′ 53″

True altitude	26° 53′ 35″			
Latitude....................	49° 30′ 0″	Secant	·187456	
Polar distance	73° 41′ 53″	Cosecant	·017835	14
Sum........................	150° 5′ 28″			
½ sum	75° 2′ 44″	Cosine	9·411816	111
½ sum ~ true altitude	48° 9′ 9″	Sine	9·872095	
			17	
			9·489219	125
			125	
			9·489094	

Aldebaran's H.A. = 19h. 30m. 8s., which is practically identical with the former result.

If the longitude is to be determined by the altitude of the moon or that of a planet, the declination and right ascension must be carefully corrected to Greenwich time, and the true altitude found from the observed altitude as is shown in Chapter XVII. The hour angle is then found as in a star chronometer, and thence the right ascension of the meridian and ship mean time.

I venture to think that the use of haversines is a distinct advantage in working out these problems. I would invite the reader to examine the two methods, and judge for himself.

ON FINDING THE HOUR ANGLE BY JOHNSON'S TABLES.

My old friend, Mr. A. C. Johnson, M.A., R.N., for many years naval instructor in H.M.S. *Britannia,* has kindly permitted me to introduce to my readers the exceedingly brief and ready method of finding the hour angle of any heavenly body by the use of a table in his pamphlet entitled "How to Find the Time at Sea in less than a Minute," published by J. D. Potter, which I strongly recommend for two very good reasons. The first is that the actual work is reduced to a minimum, and the second, that all the logs used are on one page.

Here follow the examples given above, worked by this method.

The data required are the latitude, declination, zenith distance from the observation, and the meridian zenith distance—that is, the difference between the latitude and declination if they are of the same name, or their sum if they are of different names.

THE RULE.

Take out from Table A the natural versines of the zenith distance and meridian zenith distance; their difference is the natural versine of an hour angle, the log versine of which, taken from Table B and added to the log secants of the latitude and declination Table C, give the log versine of the hour angle of the body, which must be subtracted from 24 hours if the body is east of the meridian. Tables A, B, and C, are all at the same opening of Johnson's book.

Thus in Example 1 given above we have

Latitude	49°	44′ N.
Sun's declination	16°	45′ N.
Both being north, subtract ...	32°	59′ Mer. Zen. Dist.

Zenith distance 58° 50′.

Look out in the side column Table A for 55°, and in the bottom column for 3° 45′, then in the parts column to the right for 5′, which gives us 12.

∴ Natural versine of...... 58° 50′ = 4824

Mer. zenith distance 32° 59′.

Look out in the side column Table A for 30°, and in the bottom column for 2° 45′, this gives us 1590, and then in the parts column for 14′, which gives us 23.

∴ Natural versine of...... 32° 59′ = 1613

Difference between 4824 and 1613 is

Zenith distance	58°	50′	Natural versine...	4824
Meridian zenith distance...	32°	59′	„	1613
Natural versine of 3 hours 9 minutes, Table A				3211

Latitude 49° 44′ N.

45° in side column Table C. .... } 1897
4° 45′ in bottom column Table C. ... }
1′ in parts (deduct)...... 1
∴ Log. secant of 49° 44″ .... = 1896

Declination 16° 45′ N.

15° in side column Table C. ...... } 188
1° 45′ in top column Table C. ... }
∴ Log. secant of 16° 45″ .... = 188

Log. versine of 3 hours 9 minutes, Table B.

3 hours in side column......... } 5068
9 minutes in parts column ... }
Log. versine of sun's hour angle, Table B.... 7152

And the sun's hour angle = 4 hours 5 minutes.

The time being a.m. at ship the sun is east of meridian, and this hour angle must be deducted from 24 hours.

	24h.	0m.	0s.
Sun's H.A. (E. of Mer.)......	4h.	5m.	0s.
Sun's H.A.	19h.	55m.	0s. = Ship Appt. Time

And in Example 2 we have

Latitude	49°	30′ N.
Aldebaran's declination	16°	18′ N.
Both being North, subtract ...	33°	12′ = Mer. Zen. Dist.

Aldebaran's Zen. Dist. 63° 6′

Look out in side column, Table A, for 60°, and in bottom column for 3°, this gives us 5460, then in parts column for 6′ which gives us 16.

∴ The natural versine of 63° 6′ ......= 5476

Aldebaran's Mer. Zen. Dist. 33° 12′

Look out in side column, Table A, for 33°, and in bottom column for 3°, this gives us 1613, then in parts column for 12′ which gives us 19.

∴ The natural versine of 33° 12′ = 1632
Difference between 5476 and 1632 is 3844

which is the natural versine in Table A of 3h. 28m. 0s.

Latitude 49° 30′.

45° in side column, Table C.........
4° 30′ in bottom column, Table C. } gives 1874
∴ Log. secant of 49° 30′ ...... = 1874

Declination 16° 18′.

15° in side column, Table C..........
1° 15′ in top column, Table C. ... } gives 177
Parts for 3′ 1
∴ Log. secant of 16° 18′ .... = 178

3h. 28m. 0s.

3h. 20m. in side column, Table B...
0h. 8m. in top column, Table B.... } gives 5847
∴ Log. versine 3h. 28m. 0s............ = 5847
∴ Log. versine of hour angle .... = 7899

Gives Aldebaran's hour angle..	4h.	29m.	46s. (East of Mer.)
∴ deduct it from..................	24h.	0m.	0s.
Aldebaran's hour angle	19h.	30m.	14s.

In the new navigation much use will be made of Johnson's Table on account of its brevity. It will, therefore, be well for my readers to study the above examples carefully, and by practice make themselves thoroughly conversant with it, in order to reap all the benefit to be derived therefrom.

CHAPTER XIX.

On Azimuths and Amplitudes.

In Chapter IX., it is mentioned that the deviation of the compass may be ascertained by azimuths.

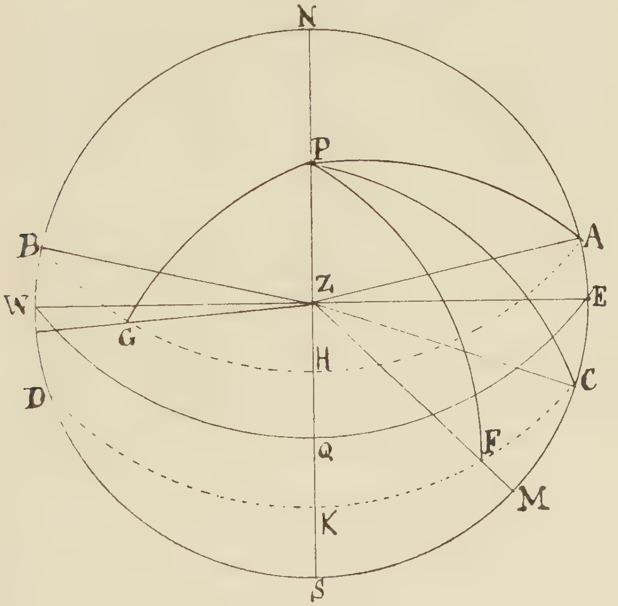

Fig. 32.

The azimuth of a heavenly body is the angle at the zenith between the circle of altitude passing through such body and the celestial meridian of the observer.

The amplitude of a heavenly body is its true bearing when it is rising or setting measured from the prime vertical, or the east or west point of the horizon.

In Fig. 32, constructed on the plane of the horizon, let P be the pole, N S the meridian of a place whose zenith is Z, W Q E the equator, W Z E the prime vertical and P A, P C, P F, and P G, portions of meridians passing through heavenly bodies whose positions are A, C, F and G.

Now the azimuth of the body at F is the angle P Z F, and that of the body at G, the angle P Z G.

And the amplitude of the body rising at A is the angle A Z E, and that of the body rising at C the angle E Z C. Also the amplitude of the body setting at B is the angle B Z W.

THE AMPLITUDE PROBLEM.

To find the true bearing of the sun at rising or setting.

In Fig. 32, when the sun is rising at A, we have in the quadrantal triangle (A Z being 90°) the two sides P A, the polar distance, and P Z, the co-latitude, to find the angle P Z A, the complement of the amplitude angle A Z E.

Now by spherical trigonometry :

$$\text{Cos. } PZA = \text{Cos. } PA \times \text{Cosec. } PZ$$

$$\text{Cos. } (90^\circ - \text{Amplitude}) = \text{Cos. } (90^\circ - \text{declination}) \times \text{Cosec. } (90^\circ - \text{latitude})$$

$$\therefore \text{Sin. Amplitude} = \text{Sin. declination} \times \text{Sec. latitude}$$

and this formula is true whether the polar distance is greater or less than 90°. For suppose the sun has south declination in north latitude, as in Fig. 32, and rises at C, then is the quadrantal triangle P C Z.

$$\text{Cos. } PZC = \text{Cos. } PC \times \text{Cosec. } PZ$$

$$\text{or Cos. } (90^\circ + \text{Amplitude}) = \text{Cos. } (90^\circ + \text{declination}) \times \text{Cosec. } (90^\circ - \text{latitude})$$

$$\text{but Cos. } (90^\circ + A) = \text{Sin. } A$$

$$\text{therefore Sin. Amplitude} = \text{Sin. declination} \times \text{secant latitude.}$$

Examples.

(1) Find the amplitude of the sun when it rose at 6 a.m. on March 31st, 1895, in latitude 51° 25′ N, longitude 5° W.

The sun's declination must be taken from the Nautical Almanac and corrected for Greenwich time; that derived from the time by the ship's clock and the longitude in time being sufficiently accurate:

Ship time	18h. 0m.	on March 30th, 1895
Longitude in time	20m.	
Greenwich time	18h. 20m.	,, ,,

Sun's declination on March 30th	3° 46′ 56″·8
58″·24 × 18·3 hours	17′ 46″·5
Sun's declination at time of observation ...	4° 4′ 43″·3 N.

$$\text{Sin. Amplitude} = \text{Sin. declination} \times \text{sec. latitude}$$
$$= \text{Sin. } 4^\circ\ 4'\ 43'' \times \text{sec. } 51^\circ\ 25'$$

Log. sin. 4° 4′ 43″	8·852082
Log. sec. 51° 25′ 0″	10·205058
	9·057140

Amplitude or sun's true bearing = E. 6° 33′ N.

The amplitude is named East because the sun is rising, and North because the declination is north.

(2) Find the amplitude of the sun when it set at 5.30 p.m., S.A.T. on April 15th, 1895, in latitude 40° 12′ S., longitude 72° 14′ E.

Ship time	5h. 30m.	on April 15th, 1895
Longitude in time	4h. 49m.	
Greenwich time	0h. 41m.	,, ,,

Sun's declination on April 15th	9° 46′ 35″·7
53″·58 × ·7 hours................................	37″·5
Sun's declination at time of observation ...	9° 47′ 13″·2 N.

Sin. Amplitude = Sin. declination × sec. latitude
= Sin. 9° 47′ 13″ × sec. 40° 12′ 0″

Log. sin. 9° 47′ 13″	9·230435
Log. sec. 40° 12′ 0″	10·117023
	9·347458

Amplitude = W. 12° 51′ 30″ N.

The amplitude is named W. because the sun is setting, and north because the declination is north.

In naming the amplitude the rule is that it is marked east if the body is rising, and west if setting, and north or south as the declination is north or south.

In Table XLII. Norie, amplitudes may be taken out by inspection.

The Altitude Azimuth Problem.

To find the true bearing of the sun by taking its altitude:

In Fig. 32 let F be the position of the sun's centre when its true altitude is F M, then P Z F is his azimuth.

In the spherical triangle P F Z, we have P F the polar distance, Z F the zenith distance, and P Z the co-latitude to find P Z F the azimuth.

The formulæ employed to solve this triangle are derived from formula (r), p. 60, without using haversines, and formula (s), p. 61, with the use of them.

From formula (s) we get:

$$\text{Hav. } PZF = \text{Cosec. } PZ \times \text{Cosec. } ZF \times \sqrt{\text{Hav. } (PZ \sim ZF + PF)} \times \sqrt{\text{Hav. } (PZ \sim ZF - PF)}$$

Or Log. hav. Azimuth = log. cosec. co-latitude + log. cosec. zenith distance + $\frac{1}{2}$ log. hav. (co-latitude ~ zenith distance + polar distance) + $\frac{1}{2}$ log. hav. ((co-latitude ~ zenith distance) ~ polar distance).

and this is the formula used almost exclusively in the Royal Navy.

From formula (r) is derived:

Sin. $\frac{1}{2}$ Azimuth = Sec. latitude × sec. altitude × cos. $\frac{1}{2}$ (latitude + altitude + polar distance, × cos) {$\frac{1}{2}$(latitude + altitude + polar distance) ~ polar distance.}

This is the formula generally adopted in the mercantile marine.

Rule I.—Using Haversines.

(1). Find Greenwich time by applying longitude in time to ship time of observation.

(2). Correct declination for Greenwich time found above and find polar distance by subtracting it from 90° if it is of the same name as the latitude, or by adding it to 90°, if they are of different names.

(3). Correct the sun's altitude in the usual manner, deduct the true altitude from 90°, and so find the zenith distance.

(4). Add together the following logs:

Log. cosec. (90° — latitude) or co-latitude
Log. cosec. zenith distance
$\frac{1}{2}$ Log. hav. (co-latitude ~ zenith distance + polar distance)
$\frac{1}{2}$ Log. hav. {(co-latitude ~ xenith distance) ~ polar distance.}

The sum of which, rejecting tens, is the log haversine of the azimuth marked the same name as the latitude, and east or west as the time is A.M. or P.M.

Example.

On June 1st, 1895, in latitude 46° 10′ N. and longitude 38° 19′ W., at 7.50 A.M. (S.A.T.) the observed alt. of the sun's L.L. was 34° 27′ 20″; I.E. + 1′ 50″; height of eye, 32 feet. Required the azimuth?

Ship Time.........	19h. 50m.	on May 31st. 1895
Long. in Time...	2h. 33m.	
Greenwich Time	22h. 23m.	,, ,,

Sun's Declination on May 31st	21°	55′	33″·9	
21″·35 × 22·4 hours.........		7′	58″·2	
Sun's Declination at time of observation.	22°	3′	32″	N.
	90°	0′	0″	
Polar Distance	67°	56′	28″	

Obs. Alt. sun's L.L..........	34°	27′	20″
I.E.		+ 1′	50″
	34°	29′	10″
Correction (23 feet)		9′	48″
True Altitude	34°	38′	58″
	90°	0′	0″
Zenith Distance..............	55°	21′	2″

Co. latitude.........	43°	50′	0″	cosec.	·159541
Zenith Distance...	55°	21′	0″	cosec.	·084790
Difference	11°	31′	0″		
Polar Distance ...	67°	56′	28″		
Sum...	79°	27′	28″	½ hav.	4·805604
Difference	56°	25′	28″	½ hav.	4·674625
				log. haversine	9·724560

Azimuth = N. 93° 28′ 45″ E.

RULE II.—WITHOUT USING HAVERSINES.

(1) Find Greenwich time as in Rule I.

(2) Correct declination, and find polar distance as in Rule I.

(3) Correct the sun's observed altitude, and find the true altitude.

(4) Add together the following logs :

Log sec. latitude
Log sec. altitude
Log cos. ½ (latitude + altitude + polar distance)
Log cos. ½ {(latitude + altitude + polar distance) ~ polar distance}

Half the sum of these logs rejecting tens is the log sine of half the true azimuth named from the south in north latitude, and from the north in south latitude.

EXAMPLE.

On February 17th, 1895, in latitude 33° 40′ S., and longitude 54° 12′ E. at 4.10 p.m., A.T.S., the observed altitude of the sun's L.L. was 34° 12′ 10″, I.E. +2′ 25″, height of the eye 21 feet. Required the azimuth ?

Ship Time	4h.	10m.	on February 17th, 1895
Longitude in Time...	3h.	37m.	
Greenwich Time......	0h.	33m.	„ „ „ „

Sun's declination at Greenwich Noon...	11°	57′	47″ ·1	Feb. 17, 1895
52·54 × ·55 hours			28″ ·9	
Sun's declination at time of observation	11°	57′	18″ ·2	S.
	90°	0′	0″	
Polar Distance	78°	2′	42″	

Obs. Alt. Sun's L.L.	34°	12′	10″
I.E.		+ 2′	25″
	34°	14′	35″
Correction (for 21 feet)		10′	24″
True Altitude	34°	24′	59″

Polar Distance	78°	2′	42″			
True Altitude	34°	24′	59″	Secant......	·083530	
					42	
Latitude.....................	33°	40′	0″	Secant......	·079732	
Sum	2)146°	7′	41″			
½ Sum........................	73°	3′	50″	Cosine......	9·464486	139
Polar Distance ~ ½ Sum	4°	58′	52″	Cosine......	9·998361	4
					9·626151	143
					143	
					2)9·626008	
				Log sine of ½ azimuth	9·813004	
					2988	
					16	

½ Azimuth ... =	40°	33′	6″
			2
Azimuth ...... =	N. 81°	6′	12″ W.

THE TIME AZIMUTH PROBLEM.

To find the true bearing of the sun at any known time: In Fig. 32, if G be the position of the sun then P G is the polar distance, P Z the co-latitude, and angle G P Z the hour angle or ship apparent time; knowing these three parts of the spherical triangle G P Z, the angle P Z A, that is the azimuth, can be found from the following formulæ in spherical trigonometry.

$$\text{Tan.}\ \tfrac{1}{2}\,(\text{P Z G} + \text{P G Z}) = \frac{\text{Cos.}\ \tfrac{1}{2}\,(\text{P G} - \text{P Z})}{\text{Cos.}\ \tfrac{1}{2}\,(\text{P G} + \text{P Z})}\,.\ \text{Cot.}\ \tfrac{1}{2}\,\text{G P Z}$$

$$\text{Tan.}\ \tfrac{1}{2}\,(\text{P Z G} - \text{P G Z}) = \frac{\text{Sin.}\ \tfrac{1}{2}\,(\text{P G} - \text{P Z})}{\text{Sin.}\ \tfrac{1}{2}\,(\text{P G} + \text{P Z})}\,.\ \text{Cot.}\ \tfrac{1}{2}\,\text{G P Z}$$

From these formulæ are obtained the values of—

$\tfrac{1}{2}\,(\text{P Z G} + \text{P G Z})$ and $\tfrac{1}{2}\,(\text{P Z G} - \text{P G Z})$
then $\tfrac{1}{2}\,(\text{P Z G} + \text{P G Z}) + \tfrac{1}{2}\,(\text{P Z G} - \text{P G Z}) = \text{P Z G}$ the azimuth.
and $\tfrac{1}{2}\,(\text{P Z G} + \text{P G Z}) - \tfrac{1}{2}\,(\text{P Z G} - \text{P G Z}) = \text{P G Z}$

and hence the following rule is derived :

THE RULE.

(1.) Find Greenwich time by applying longitude in time to ship time.

(2.) Correct declination and equation of time for the above Greenwich time, and find polar distance.

(3.) Find ship apparent time from chronometer time by applying to it the error of chronometer longitude in time and equation of time. If a.m. at ship deduct it from 24 hours, which will give the polar angle; if p.m. at ship, the ship apparent time is the polar angle.

4. Add together the following logs :

$$\left\{\begin{array}{l}\text{Log. cosine}\ \tfrac{1}{2}\ (\text{Polar distance} \sim \text{co-latitude})\\ \text{Log. secant}\ \tfrac{1}{2}\ (\text{Polar distance} + \text{co-latitude})\\ \text{Log. cotangent}\ \tfrac{1}{2}\ \text{Polar angle}\end{array}\right.$$

The sum of these is the log. tangent of ½ (Azimuth + P G Z) (a).

Add together also the following logs:

Log. sine ½ { Polar distance ~ co-latitude)
Log. cosec. ½ (Polar distance + co-latitude)
Log. cotangent ½ Polar angle.

The sum of these is the log. tangent of ½ (Azimuth — P G Z) (b).

Then the sum of these two angles (a) and (b) gives the azimuth if the polar distance is greater than the co-latitudes. But it is their difference if the co-latitude is the greater; because in a spherical triangle the larger angle is always subtended by the larger side.

Example.

On May 19th, 1895, at 7·45 a.m. (S.A.T.) in latitude 42° 20′ N. and longitude 150° 15′ W., a chronometer showed 6h. 30m. 41s. whose error on G.M.T. was 33m. 22s. fast. Required the azimuth.

Time at Ship.........	19h. 45m. on May 18th, 1895.
Longitude in Time	10h. 1m.
Time at Greenwich	5h. 46m. on May 19th, 1895.

	Sun's declination.		Equation of Time.
At Greenwich, noon 19th	19° 46′ 34″ ·3	....	3m. 45s. ·28
32″ ·16 × 5·8 hours	3 6″ ·5	$\overset{\text{s}}{\cdot 1}$12 × 5·8 hrs.	·65
At time of observation ...	19° 49′ 41″ N.	....	3m. 44s. ·63+
	90° 0′ 0″		
Polar Distance	20° 10′ 19″	to be added to mean time.	

Chronometer showed	6h. 30m. 41s.
Error on G.M.T. ...	33m. 22s. fast.
G.M.T..................	5h. 57m. 19s. on May 19th
Long. in Time	10h. 1m. 0s.
S.M.T..................	19h. 56m. 19s. on May 18th.
Equation of Time ...	+ 3m. 45s.
S.A.T..................	20h. 0m. 4s. „ „
	24h. 0m. 0s.
Polar Angle	3h. 59m. 56s.
½ Polar Angle	1h. 59m. 58s.

Polar Distance	70° 10′ 19″				
Co-latitude ...	47° 40′ 0″				
Sum	117° 50′ 19″				
Difference......	22° 30′ 19″				
½ Sum	58° 55′ 10″	secant ...	·287164	cosecant ...	·067296
½ Difference...	11° 15′ 10″	cosine ...	9·991568	sine	9·290394
½ Polar Angle	1h. 59m. 58s.	cotangent	10·238707	cotangent..	10·238707
		Log. tangent (a)	10·517439	Log. tangent (b)	9·596397

(a) = 73° 6′
(b) = 21° 32′ 45″

Azimuth = N. 94° 38′ 45″ E. = S. 85° 21′ 15″ E.

(a) and (b) are added together because the co-latitude is less than the polar distance, and therefore the azimuth, the angle opposite to the latter, must be the greater angle.

On Burdwood's and Davis's Azimuth Tables.

To ascertain the azimuth by means of calculation involves a considerable amount of time, which can often ill be spared by the navigator at sea. In the above mentioned tables the sun's azimuth is given for every four minutes of apparent time, and to each degree of declination, and to the degrees of latitude from the equator to 30° N. & S. in Davis's, and from 30° to 60° N. & S. latitude in Burdwood's.

Take the last example worked out above. We have :

Time by Chronometer ...	6h. 30m. 41s.
Error Fast	33m. 22s.
G.M.T.	5h. 57m. 19s. May 19th.
Long. in Time	10h. 1m. 0s. W.
S.M.T.	19h. 56m. 19s. May 18th
Equat. Time	3m. 45s.
S.A.T.	20h. 0h. 4s. or 8 a.m.

Apparent Time at Ship	8 a.m.
Latitude	42° 20′ N.
Declination	19° 49′ N.

Latitude and Declination being of the same name, at p. 125 in Burdwood's Tables we find the nearest latitude 42°, and in the column under the nearest declination 20°. Opposite 8 a.m., on the left, we find the azimuth to be 94° 13′, and by the directions given at the bottom of page 124 we name it N. 94° 13′ E.

No further explanation is required. Care, however, must be taken to look out the azimuth in its proper place, and, when the latitude and declination are of contrary names, that it is not taken from the page on which they are of the same name.

A very serious defect in these tables is that they do not give azimuths of the sun when his altitude exceeds 60°. Thanks, however, to the scientific researches of modern navigators, we are able very easily to supply this deficency.

* BLACKBURNE'S A. AND B. AZIMUTH TABLES.

I am enabled, by the courtesy of Capt. Blackburne, to place before my readers an explanation of his very valuable tables for finding the azimuth of the sun and that of stars whose declination does not exceed 23° 28′, as also that of Vega, Capella, α Crucis, and Polaris, whose declinations exceed that amount. (A recent publication, "The Bearings of the Principal Bright Stars," by Mr. A. C. Johnson, M.A., R.N., recently issued, is most useful in this regard).

The requisite data are the hour angle within 4 minutes, the latitude and the declination.

RULE.

Enter Table A with the hour angle and latitude, and Table B with the hour angle and declination. The correction from Table A is always + (plus) except when the hour angle exceeds six hours, then it is – (minus)· The correction from Table B is – (minus) when latitude and declination are of the same name, and + (plus) when they are of different names. The sum or difference of these corrections gives us a diff. long. correction for 1′ of altitude. Turn this diff. long. correction into departure by the traverse table or by Table I. in Capt. Blackburne's pamphlet. Then in Table II. this departure gives us the degrees and tenth of a degree of the heavenly body's azimuth, which is named in North latitude from the South for a + correction, and from the North for a – correction. And *vice versâ* for South latitude.

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* Published by R. H. Laurie, 35, Minories, London, E.

The above seems rather intricate, but it is very easy to work, and by it we are able to get the azimuth of heavenly bodies at all altitudes and from 60° N. to 60° S. latitude.

Examples.

On April 20th, 1895, at about 9.10 a.m. in latitude 47° 15′ N. and longitude 13° 45′ W., when a chronometer showed 10h. 6m. 10s., whose error on G.M.T. was 40 seconds fast, the sun bore by compass S 41° E. Find the compass error?

Chronometer showed.........	10h.	6m.	10s.
Error		fast	40s.
G.M.T. April 19th...........	22h.	5m.	30s.
Longitude in Time...........		55m.	0s.
S.M.T. April 19th	21h.	10m.	30s.
Equat. of Time	+	1m.	7s.
S.A.T. April 19th	21h.	11m.	37s.
	24h.	0m.	0s.
Sun's H.A. (East of Mer.)...	2h.	48m.	23s.

Examples.

1.

S.A.T. = 21h. 11m. 37s. Lat. 47° 15′ N. Declination 11° 30′ N.

H.A. = 2h. 48m. 23s. Table A Lat. 47° N. + 1·191
" " " B Dec. 11½° N. − ·304
Diff. long. correction + ·887
Lat. 47°. Departure = + ·605
Azimuth = S. 58° 48′ E.
Bearing by compass S. 41° E.
Compass error 17° 48′ W.

Burdwood's Azimuth Tables give us the azimuth as N. 121° 9′ E. by averaging for the declination 11° 30′ N., and this is equal to S. 58° 51′ E. A is + because the hour angle is less than 6 hours, and B is — because the latitude and declination are of the same name. The azimuth is named from S. to E. because the latitude is N. and the diff. long. correction is + and, the time being a.m., the sun is east of meridian.

2.

On March 25th, 1895, at about 4h. 40m. p.m., in latitude 57° 30′ S. and longitude 79° 47′ W., when a chronometer showed 11h. 15m. 31s., whose error on G.M.T. was 1h. 10m. 11s. fast. Find the sun's true azimuth.

Chronometer showed	11h. 15m. 31s.
Error	1h. 10m. 11s. fast
G.M.T. March 25th	10h. 5m. 20s.
Longitude in Time	5h. 19m. 8s. W.
S.M.T.	4h. 46m. 12s.
Equation of Time	— 5m. 57s.
S.A.T.	4h. 40m. 15s.

S.A.T. = 4h. 40m. p.m. Lat. 57° 30′ S. Declination 2° N.

H.A. 4h. 40m.	Latitude 57½° S.	Table A + ·571
,, ,,	Declination 2° N.	,, B + ·037
		Diff. long. correction + ·608
		Departure = + ·327
		Azimuth = N. 71° 54′ W.

which agrees with Burdwood. In this case the latitude is south, the diff. long. correction +, and the sun west of meridian; the azimuth is therefore named from N. to W.

3.

On May 28th, 1895, at about 1 a.m., in latitude 35° 0′ S. and longitude 10° 0′ W. Find the true azimuth of Vega.

Ship Time	13h. 0m. on the 27th
(Roughly) R.A.M.S.	4h. 22m.
R.A. Mer.	17h. 22m.
R.A. Vega	18h. 33m.
H.A. Vega	22h. 49m. (W. of Mer.)
„ „	1h. 11m. (E. of Mer.)

H.A. Vega 1h. 11m. (E. of mer.). Lat. 35° 0′ S. Declination 38° 41′ N.

H.A. 1h. 11m.	Latitude 35° 0′ S.	Table A = + 2·200
„ „	Declination 38° 41′ N.	„ B = + 2·620

Diff. long. correction + 4·820
Corresponding Departure = + 3·94
Azimuth = N. 14° 12′ E.

In the foregoing examples the hour angle of the sun is the ship apparent time if the sun is west of meridian, but 24 hours less the ship apparent time if it is east of meridian, which can be ascertained by the chronometer time being corrected for error of chronometer, longitude in time, and equation of time.

The hour angle of any star is found by correcting the chronometer time of observation for error of chronometer and longitude in time, thus finding ship mean time, which added to the right ascension of the mean sun, gives sidereal time at ship; from this the star's hour angle is obtained by deducting its right ascension.

The object of finding the true bearing of the sun by the azimuth and amplitude problem, as stated in Chapter IX., is to ascertain the compass error and deviation. Thus if in the first example of the amplitude problem at p. 191 the sun bore by compass when rising E. b. S., and the variation was 24° W., we have :

Sun's true bearing	E. 6° 33′ N.
or	N. 83° 27′ E. right
Variation...........	24° W. right
Sun's magnetic bearing......	N. 107° 27′ E.
or	S. 72° 33′ E.
Sun's compass bearing	S. 78° 45′ E. = E. b. S.
Deviation	6° 12′ E.

Or it can be worked in the following manner :

Sun's true bearing	N. 83° 27′ E. right
Sun's compass bearing...	S. 78° 45′ E. left
or	N. 101° 15′ E. right
Total error or compass correction	17° 48′ W.
Variation	24° 0′ W.
∴ Deviation......	6° 12′ E.

because the compass correction consists of the variation and deviation combined.

In the second example of the same problem, if the sun bore by compass when setting W. b. S., and the variation was 10° E., we have

Sun's true bearing......	N. 77° 8′ 30″ W. = W. 12° 51′ 30″ N.
Sun's compass bearing	N. 101° 15′ 0″ W. = W. b. S.
Total error or compass cor.	24° 6′ 30″ E.
Variation...	10° 0′ 0″ E.
∴ Deviation	14° 6′ 30″ E.

In finding the compass correction, variation, or deviation by comparing true, magnetic, or compass bearings, it is imperatively necessary to remember the following rules :

(1) If the true bearing is named to the *right* of the compass bearing, the *compass correction* is East.

(2) If the true bearing is named to the *left* of the compass bearing, the *compass correction* is West.

(3) If the true bearing is named to the right of the magnetic bearing, the *variation* is East.

(4) If the true bearing is named to the *left* of the magnetic bearing, the *variation* is West.

(5) If the magnetic bearing is named to the *right* of the compass bearing, the *deviation* is East.

(6) If the magnetic bearing is named to the *left* of the compass bearing, the *deviation* is West.

Of course the bearing of any object is the same; it is differently named because the compass needle is affected by variation, and, in iron ships, by deviation.

Perhaps Fig. 33 may serve to explain more clearly the foregoing statement. It represents a compass card whose north point is true north. Let M be the magnetic north point, or the position which the north point of the compass would assume if there was no local attraction, as in a wooden ship, and D the compass north, or the position it actually does take. Now, if an object bears west (true), it will evidently bear N.W. b. W. (magnetic), and W. b. N. (compass); the variation is 3 points west, and the deviation is 2 points E. Now we have the true bearing west, the magnetic bearing N.W. b. W., the former 3 points to the *left* of the latter ; and the compass bearing W. b. N., the true bearing being 1 point to the left of it, giving the compass correction as 1 point W., and the magnetic bearing, W. b. N. being 2 points to the right of the compass bearing, N.W. b. W., giving the deviation as 2 points E., which the card shows to be correct.

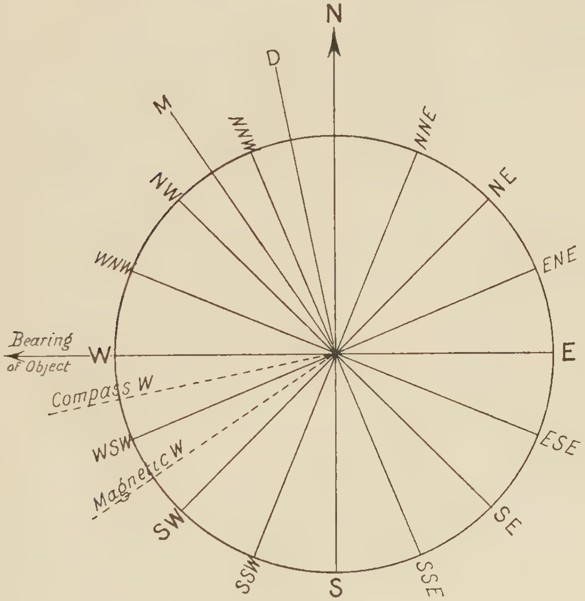

N
D
M
NNW
NNE
NW
NE
WNW
ENE
Bearing of Object
W
E
Compass W
WSW
ESE
Magnetic W
SW
SE
SSW
S
SSE

Fig. 33.

This is, of course, always true, and can easily be verified.

CHAPTER XX.

ON THE USE OF CHARTS.

A CHART is the representation of a portion of the earth's surface drawn on the flat. It is impossible without distortion to do this, and different methods or projections are in use. The only one, however, which we will consider is that known as Mercator's projection. Charts were first drawn in this manner by Gerard Mercator in 1569.

In a Mercator's chart the meridians of longitude are drawn parallel to each other. Now we know that on the earth's surface meridians one degree apart are sixty geographical miles distant from one another at the equator, and that they gradually converge until at either pole they meet. It is evident, therefore, that, except at the equator, the meridians on a Mercator's chart will be too far apart. For instance, in latitude 60° meridians one degree apart are only thirty miles distant from one another, on the earth, and this distance vanishes altogether at either pole. We are also aware that the distance between parallels of latitude one degree apart is *always* sixty geographical miles. I am, of course, neglecting the slight difference which is due to the earth not being a perfect sphere. In a Mercator's chart the distances are increased between the parallels of latitude in exactly the same proportion as the distances between the meridians have been enlarged in order to draw them parallel to each other.

The great advantage derived from the use of such a chart is that the course of a ship, which, as we have seen, is the angle between the ship's track and the meridian—in reality a curve—can be laid down as a straight line. Of course, in the polar regions the distortion becomes enormous, and Mercator's charts of the pole and its surroundings are incapable of construction, because at the pole the meridians meet, and the distance between the parallels of latitude becomes infinitely great, therefore the proportion cannot be preserved graphically. Gnomonic projection is used for the construction of polar charts.

There are two descriptions of Mercator's charts in use, known as "true" and "magnetic" charts.

In a true chart the compass cards drawn on it have the north point of the needle directed to the north point of the earth, that is, the north and south points of the compass needle coincide with the meridian.

In a magnetic chart the north point of the needle in its compass cards is directed to the magnetic pole of the earth. As the magnetic variation is gradually altering, old magnetic charts may be considerably in error; the variation in the English Channel, for instance, has changed about a quarter of a point during the last twenty years. It is, therefore, best to use a true chart and apply the correct variation. Magnetic charts are, however, generally supplied, and therefore I give one as an example.

Plate No. III. is a magnetic chart drawn on Mercator's projection, extending from 57° 52′ 20″ N. to 57° 19′ 30″ N. latitude, and from 6° 51′ 40″ W. to 8° 0′ 0″ W. longitude.

When using any chart, it should be placed on a flat surface, with its south end next to you. The meridians run from top to bottom of the chart, and the parallels of latitude at right angles to the meridians, or from side to side of the chart. On the margin to the right and left is a graduated meridian, divided into degrees and minutes of latitude. At the bottom or nearest side of the chart, and also at the top, the degrees of longitude are each divided into sixty minutes of longitude, *which are nowhere equal to geographical miles except on the equator.* It is therefore evident that, as the nautical mile on the equator in a Mercator's chart is equal in length to a minute of longitude on the equator, but that it gradually increases in length as it gets further from the equator till at either pole it becomes infinitely great, it would be incorrect to measure distances in nautical miles at the top or bottom of such a chart. But the minutes of latitude are made larger in proportion, and are therefore always equal to a nautical mile on their particular parallel, and therefore great care should be taken that *all distances be measured on the graduated meridians at the side of the chart in a line with such distances.* On this chart are drawn a parallel of 57° 30′ N. latitude and the meridians of 7° 0′ 0″ and 7° 30′ 0″ W. longitude.

(1) The first problem with which I shall deal is how to place upon a chart a point whose latitude and longitude are known. The latitude is measured on the side of the chart, and the longitude at the top or bottom. For instance, we want to find the position of a point in latitude 57° 46′ N. and longitude 7° 50′ W. The two lines A B and A C are respectively parallel to the parallels of latitude and to the meridians. A B is equal to 10′ of longitude measured at the

top or bottom, and A C is equal to 16′ of latitude measured at either side, and these distances are taken, one from 8° 0′ W. longitude to the eastward, the other from 57° 30′ N. latitude to the northward, and the latitude of A is 57° 30′ plus 16′ = 57° 46′ N., and the longitude of A is 8° 0′ less 10′ = 7° 50′ W., therefore A is the point required. A good plan is to use parallel rulers for laying off the latitude when the point required is not too far from the edge of the chart, or from a graduated meridian, and the dividers for measuring off the longitude along the edge of the parallel ruler.

(2) To find the course and distance from one place to another on the chart.

Lay the edge of the parallel ruler between the two points, then move it to the compass card on the chart, taking care not to let it turn in doing so; the magnetic course is indicated by the edge of the ruler on the compass card. Put the points of the dividers on the respective places, then at the side of the chart *opposite to them* find how many miles they are apart.

Required the magnetic course and distance from N. Loch to Lower Shillay. It is E $\frac{1}{4}$ S magnetic fourteen and a quarter miles.

(3) Being given the course and distance from a known position, to find the latitude and longitude of the place arrived at.

With the parallel rulers take the given course from the compass on the chart and lay it off from the point named, then with the dividers measure the distance at the side of the chart, and lay it off along the course from the point of departure, then find the latitude and longitude of the point thus reached by measurement from the nearest parallel of latitude and meridian of longitude.

A ship sails from the Griminish Point N.N.W. magnetic fifteen miles, required the latitude and longitude of the point arrived at. In the chart lay off the course with the parallel rulers and open the dividers to thirty-two miles at the side of the chart opposite, then measure this distance along the course laid down, when D will be found to be the point arrived at, whose latitude and longitude are 57° 49′ 40″ N., and 7° 51′ W.

In these problems we have been dealing with magnetic courses only, and if in a wooden vessel, you can at once order the helmsman to steer such a course. But the ship may be wholly or partly constructed of iron, and the compasses will be affected by the magnetism of the ship. It would be incorrect then to tell the helmsman to steer a magnetic course; the deviation must be allowed in order to find the compass course. For this purpose we will use the following portion of a deviation card, showing magnetic courses corresponding to compass courses, which we shall find very convenient.

Ship's Head by Standard Compass.	**Deviation.**	Corresponding Magnetic Course.
North	2° E.	N. 2° E.
N. by W.	1° W.	N. 12 W.
N.N.W.	4° W.	N. 26½° W.
N.W. by N.	7° W.	N. 40¾° W.
N.W.	14° W.	N. 59° W.
N.W. by W.	16° W.	N. 72¼° W.
W.N.W.	15° W.	N. 82½° W.
W. by N.	12° W.	S. 89¼° W.
West	6° W,	S. 84° W.
W. by S.	4° W.	S. 74¾° W.
W.S.W.	2° W.	S. 65½° W.
S.W. by W.	2° E.	S. 58¼° W.

The variation is 24° 50′ W.

The course from Griminish Point to D. is N.N.W. magnetic, which is equal to N. 22½° W., the deviation from the above table, as the magnetic course lies between N. 26½° W. and N. 12° W., is about 3° W., which makes the compass course N. 19½° W., and we should tell the helmsman to steer N. by W. ¾ W. It will be observed that to find a compass course from a magnetic course, it is necessary to average the deviation, because that is determined *for the ship's head by compass.* It is a good plan to write down the corresponding magnetic courses as above, as it makes the work of averaging much more easy. But it is better still to use a Napier's diagram, which prevents any confusion or error.

The position of a ship at H, Plate III., is determined by the two bearings, called cross-bearings, of Griminish Point and Haskeir Island, which are East and N. by E., correct magnetic, respectively. In the case of an iron ship the bearings taken by the standard compass must be corrected, for the deviation due to the magnetism induced and which varies according to direction of the ship's head. Let us suppose the direction of the ship's head in this case to have been N.W. by W., the deviation on this point is 16° or 1½ points W., and these bearings would have been E. by S.½S. and N.N.E.½E. by ship's standard compass. The best cross bearings are those which are 90° apart.

An analogous method to the preceding is that by which a ship's position at G, Plate III., is found. The compass bearing of a known point, such as is, in this case, Haskeir Island, is taken; and at the same time the angle between this point and another is observed with the sextant. By ship's standard compass Haskeir Island bears E.S.E., the ship's head being W. b. N., and at the same time the angular distance between Haskeir Island and Shillay Lighthouse by the sextant is 72°. *The line of magnetic bearing,* E. by S. (see deviation table, p. 212), is ruled off on the chart, and with a protractor on any part of this line, the point F for example, the angle 72° is laid off; where a line drawn parallel to that forming the angle with the line of bearing, passing through the position of Shillay Lighthouse, cuts the line of magnetic bearing of Haskeir Island, is the position required.

A very useful way to find the distance of a ship from any headland or other well defined object on the coast that she may be passing, is the operation known as "doubling the bow angle."

Suppose a ship to be steering north, and that some conspicuous object on the shore bears N.E., the bow angle, or the angle between the direction of the ship's head and that of the object is 4 points or 45°: now, if the ship continues on the same course, north, till the object bears 8 points or 90° from the bow—that is in this case east—she will have doubled the bow angle, and her distance from the object observed will be exactly the same as the distance she has run in the interval between the two observations. Course and bearings by standard compass are to be employed, and the distance at the second observation will be correct. But in order to lay off the bearing on the chart to find the ship's position at the time of the second observation, the deviation due to the ship's course must be applied.

Thus, to determine the position of a ship steering N.N.E. by compass (deviation 2 points E.), passing to the westward of the Monach Islands, the Shillay Lighthouse was observed to bear N.E. by E., that is, 3 points on the bow, and, after running five miles, it bore east, that is, 6 points on the bow; thus, having doubled the bow angle the distance of the ship from the Shillay Lighthouse was five miles, the distance run between the observations, and it bore east by compass, which, with 2 points E. deviation, is E.S.E. magnetic. The ship, therefore, is five miles from Shillay Lighthouse, which bears E.S.E. magnetic, and this is laid off on the chart at K. I have also drawn on Plate III. the magnetic course and bearings in order to show geometrically that the triangle formed by the course of the ship and the two bearings is an isosceles triangle, the distance run and distance from the point at the last observation being the equal sides, because the two angles at the base, that is the distance of the ship from Shillay Lighthouse at the first observation, are equal to each other. The angle in the triangle at K being equal to 180° less 6 points or 67½°. The other two angles must, therefore, each equal 3 points, or 33¾°.

To Find the Position of a Ship by Angular Measurement.

If the angular distance between two points is observed, the information obtained is that the ship is somewhere upon the circumference of a circle at whose centre the above two points subtend double the angle observed. Therefore, if the angles between three points on a coast line be observed, it is easy, without using the compass, to find the position of the ship by the intersection of the arcs described, as shown on Plate III. There are practically two cases involved: the first is when the observed angle is less than 90°, and the second when it exceeds that angle.

FIRST CASE.—WHEN THE OBSERVED ANGLE IS LESS THAN 90°.

To describe the circle: Join the two points between which the angle is taken, then lay off with a protractor from each end of the line joining the two points, towards the ship, an angle equal to the complement of the observed angle—that is what it wants of 90°.

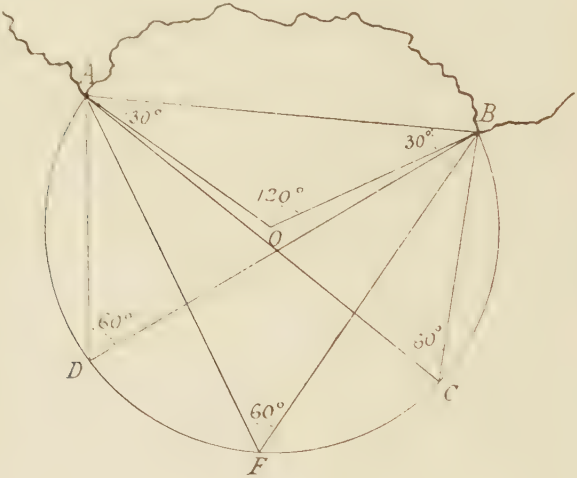


FIG. 34.

In Fig. 34, let A and B be two conspicuous points, and the angle subtended between them at the ship be 60°. Join A B: with a protractor *lay off towards the ship* the angles O B A and O A B, each 30°, with their point of intersection, O, as a centre, and radius O A or O B, describe part of a circle A D F C B. The ship must be somewhere on this arc. Because by Euclid, III., 20, *the angle* *at the centre of a circle is double the angle at the circumference on the same base, that is, on the same arc.* The angle A O B is equal to 120°, because the three angles of any plane triangle are together equal to 180°, and the angles O A B and O B A are together equal to 60°. The angle at the ship is 60°, that is one half of the angle at the centre O subtending the base A B.

SECOND CASE.—WHEN THE ANGLE OBSERVEL EXCEEDS 90°.

In Fig. 35, let the angle observed from the ship between the points A and B be 130°. Join A B. Lay off from each end of this line, at A and B, *on the opposite side to the ship*, the angle by which the observed angle exceeds 90°, viz., 40°. Angles O B A and O A B are each 40°. With their point of intersection O as a centre, and O A or O B as a radius, describe part of a circle C A D E B F. The ship is somewhere on the arc A D E B.

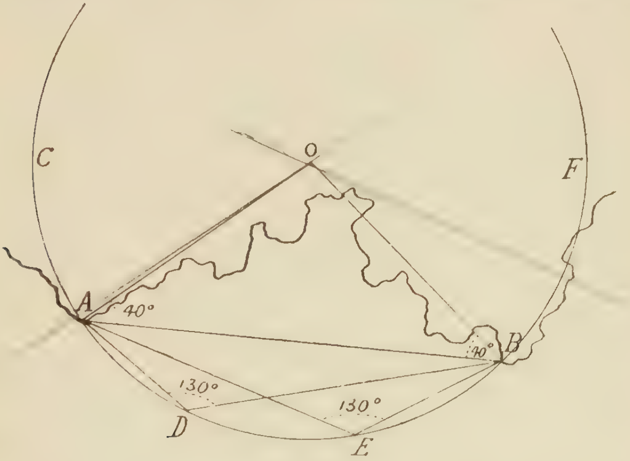


FIG. 35.

The protractor will show the truth of these demonstrations, which are founded on Euclid III., 20, 22, and 31; and that the angles A D B, A E B, or any others subtended by lines drawn from A and B, meeting at any point in the arc A D E B, will subtend an angle of 130°, which was that observed from the ship.

In Plate III. the angles from a ship have been observed between Shillay Lighthouse and Bo Ruag, and between Bo Ruag and A. Chuile, to be 56½° and 60° respectively. The circles due to these angles have been drawn, and it is evident that their point of intersection M must be the position of the ship at the moment the two angles were observed.

In a similar manner, N is the position of a ship from whose deck the angles between Weaver Point and Michael Point, and between Michael Point and Gob-na-Caiplich, were 40° and 100° respectively.

It is very useful, when beating up a rocky coast where there are outlying dangers, to understand the principle of *the danger angle.*

In Plate III. Bo Ruag is a rocky patch submerged at high water. Suppose a vessel is beating up against a N.W. wind, and it is desirable that she should not go within a mile of this danger.

Draw a circle with the rock as a centre and the length of a mile as a radius round the rock; then draw a line between two prominent points on each side of the danger, such as Shillay Lighthouse and A. Chuile; from the centre of this base and shorewards erect a perpendicular; find, by trial, the spot on this perpendicular line at which one compass leg must be placed to draw a circle passing through Shillay Lighthouse and A. Chuile, which just touches the circumference of the small circle drawn round the danger; draw this circle and find out by the protractor what angle is subtended from any part of its circumference *outside the danger* by the two points mentioned. In this case it is 147°, and this is the danger angle. Put this angle on the sextant and see that the ship in standing inshore never gets so close in that the angle subtended by the above two points from her deck exceeds 150°; she will then never approach within less than a mile of the dangerous ground.

PRACTICAL CHART WORK.

On Plate IV. the track of a ship rounding Ushant is laid down, due to the following sailings. The deviations used are these upon which the curve in Napier's diagram Plate I., is described. All the courses and bearings given are by the standard compass, and they, therefore, require to be converted into magnetic courses and bearings before they can be placed on the chart.

At 4 a.m. the ship, steering N.N.E. the deviation on this course by the Napier's diagram, Plate I., is 5° E.

The Ar Men Rock Light bore	S.E.
The Ushant Light bore	N.E. ½ N.

These are cross bearings, and must be corrected for the deviation due to the position of the ship's head before being marked on the chart.

Compass bearing Ar Men Rock Light S.E.	=	S. 45° E.
Deviation due to ship's head N.N.E.	=	5° E.
Magnetic bearing Ar Men Rock Light	=	S. 40° E.

Compass bearing Ushant N.W. Light N.E. ½ N. = N. 39° E.
Deviation due to ship's head N.N.E. .... = 5° E.
Magnetic bearing Ushant N.W. Light = N. 44° E.

Ship's course, by standard compass, N.N.E. .... = N. 22° 30′ E.
Deviation due to this course .... = 5° E.
Ship's course (magnetic) .... = N. 27° 30′ E.

Put over Patent log.

At 5 a.m. Patent log showed 8 knots.

Ushant N.W. Light bore N.E. ½ E. .... = N. 50° E.
Deviation due to ship's head N.N.E. .... = 5° E.
Magnetic bearing .... = N. 55° E.

The bow angle with regard to the bearing of Ushant N.W. Light is 2½ points by compass. When, therefore, this is doubled, that is, when Ushant N.W. Light bears E. by N., the distance run since 5 a.m. will be the distance of the ship from the lighthouse.

At 5.50 a.m. Ushant N.W. Lighthouse bore E. by N.; the Patent log showed 6½ knots.

Compass bearing Ushant N.W. Lighthouse E. by N. = N. 79° E.
Deviation ship's head N.N.E. .... = 5° E.
Magnetic bearing Ushant N.W. Lighthouse = N. 84° E.

which is distant 6½ miles.

Altered course to E. ¾ N. Deviation 16° W.

At 6.25 a.m. Ushant N.W. Lighthouse abeam.

At 7 a.m. Ushant N.W. Lighthouse bore S.W. ¼ S. Ushant N.E. Lighthouse bore S. ¾ E.

Compass bearing Ushant N.W. Lighthouse S.W. ¼ S. = S. 42° W.
Deviation ship's head E. ¾ N. = 16° W.
Magnetic bearing Ushant N.W. Lighthouse = S. 26° W.

Compass bearing Ushant N.E. Lighthouse S. ¾ E. ... = S. 8½° E.
Deviation ship's head E. ¾ N. .... = 16° W.

Magnetic bearing Ushant N.E. Lighthouse .... = S. 24½° E.

Altered course to E. ¾ S. Deviation on this course 20° W.

Compass course E. ¾ S. .... = S. 81½° E.
Deviation on this course...................................... = 20° W.

Magnetic course .... = S. 101½° E.
= N. 78½° E.

At 9 a.m. Le Four Lighthouse bore S.W. ¾ W., and the angular distance between Le Four Lighthouse and Vierge Lighthouse was observed with the sextant to be 96°.

Compass bearing Le Four Lighthouse S.W. ¾ W. ... = S. 53½° W.
Deviation ship's head E. ¾ S. .... = 20° W.

Magnetic bearing Le Four Lighthouse .... = S. 33½° W.

All courses taken from the chart are correct magnetic, and it is necessary to find the deviation on the Napier's curve due to such magnetic course, and apply it to the magnetic course in order to obtain the course to steer by compass. Thus at 4 a.m. above, it was desired to steer N. 27½° E. magnetic, and on the Napier's curve the deviation on that course measured from N. 27½° E., parallel to the plain lines to the curve is found to be 5° E., and this applied to the magnetic course to the left gives N. 22½° E. or N.N.E., the course to be steered by compass.

In wooden vessels, having no deviation, chart work is much more simple; the course measured on the chart is that to be steered by the ship's compass.

Marks on Charts.

The small numbers which are dotted over the water surface of a chart are the soundings, that is the depth of water at low water ordinary spring tides. They are in fathoms or in feet as indicated on the chart. When there is a line and a dot above the number, thus, $\dot{\overline{250}}$, it means that at 250 fathoms no bottom was found.

The small letters under the sounding indicate the nature of the bottom, such as s., sand; sh., shells; m., mud; &c.

The letters H.W.F. and C. mean high water full and change; that is high water at the place on the day of new and full moon, the time being indicated thus: IX. 19, meaning nineteen minutes past nine after noon on those days. The average rise of the water above the mean low water level of ordinary spring tides at spring, and neaps is also often given.

Currents are sometimes represented by arrows, which when feathered represent the flood stream; they point in the direction in which the stream runs; close to the arrow is a number, which indicates the velocity of the current in knots or nautical miles.

Lights are represented by a red spot surrounded by a patch of yellow, and their nature by abbreviations; such are Lt.F., light fixed; Lt.Rev., light revolving, &c.

Dotted lines surrounding the coast lines at different distances are lines of equal soundings.

My readers will learn more by studying a chart for a short time than volumes of description will teach them.

CHAPTER XXI.

The New Navigation or Double Chronometer Problem.

Upwards of one hundred years ago the principle upon which this method of ascertaining a ship's position was recognised, and, moreover, it was put into practice to a limited extent by officers in the old East India Company's ships, and by a few others. Owing to the inaccuracy of the timekeeper in those days, though the latitude was correctly determined by its means, the longitude obtained was only an approximation. The introduction of the modern chronometer, however, has totally changed the situation, and the solution of the double chronometer problem enables the mariner to find the exact position of his ship at almost any time of the day or night. It may, therefore, be safely asserted that, as the "New Navigation" becomes better known, it will take its proper place as the most convenient and accurate way of navigating a ship and be universally practised.

In 1843 Captain Thomas H. Sumner, of Boston, U.S.A., published a work entitled "A New and Accurate Method of Finding a Ship's Position at Sea, by projection on a Mercator's chart. When the latitude, longitude, and apparent time at ship are uncertain, one altitude of the sun with the true Greenwich time determines: first, the true bearing of the land; secondly, the errors of longitude by chronometer, consequent to any error in the latitude; thirdly, the sun's true azimuth. When two altitudes are observed, and the elapsed time noted, the true latitude is projected; and if the times be noted by chronometer, the true longitude is also projected at the same operation."

I give the title in full because it explains what the Sumner problem can determine provided that all the data are correct, the latitude and longitude of the course being only approximately known.

The following is a brief exposition of the principle involved:

If an altitude of the sun, or that of any other heavenly body is observed, and the true bearing or azimuth of such body is known, at all places on a line at right angles to such bearing for a considerable distance, its altitude will be practically the same, and, therefore, we may affirm that the ship is somewhere on this line, which is termed very appropriately a "line of position." Another "line of position" can be obtained by observing the altitude of some other heavenly body at the same time. It is best that the bearings of the two bodies should be as nearly as possible at right angles to each other, certainly they should never vary more than 45° from that position; that is to say, that the angle between their respective bearings should lie between 45° and 135°. Or a second altitude of the same heavenly body may be used when a sufficient time has been allowed to elapse to cause its bearing to change sufficiently to give a sufficiently large angle. The two "lines of position" thus found are laid off on the chart, and their point of intersection must be the ship's position.

The method of drawing these "lines" is fully and practically explained in the examples given.

Under the old system, in working out a chronometer observation, a latitude by dead reckoning must be used which is most probably erroneous. The best latitude for this purpose is derived from the meridian altitude of the sun, but then the run of the ship has to be allowed from or to the time of sights, and this may, and very often is, incorrectly registered. Even the patent log only shows how far the ship has gone through the water; it cannot indicate any current which may have set her miles out of her course in the interval; and if the latitude used is wrong, the longitude obtained is incorrect.

Under the new system it is not necessary to wait till noon for the latitude. If the horizon is clear and two stars in good relative positions are observed we can fix our place at any time of the night. A most useful way of using this method is to take an altitude of a star whose bearing is about S.W. just before sunrise, when the horizon is usually very distinct, and combine it with an altitude of the sun taken at the customary time of morning sights, the ship's position can then be determined with great accuracy providing that the run of the ship in the interval has been most carefully registered.

In the examples given the work is abbreviated and simplified by the use of Johnson's, Blackburne's, and Burdwood's Tables, a full explanation of the use of which has been already given. The hour angles can, of course, be found in the usual manner. The method of working here used, however, is the shortest and simplest extant.

Let us first take the case of two or more simultaneous altitudes of different heavenly bodies. The chronometer gives the exact Greenwich mean time (G.M.T.) of the observations for which the different data from the "Nautical Almanac" are corrected. With the dead reckoning (D.R.) latitude we find the respective hour angles of the bodies observed, and hence the mean time at ship (S.M.T.) and longitude. If the longitude thus found is the same for each heavenly body observed, then it is correct, and the latitude used is also correct. But if they do not agree, the longitudes found are laid off on the chart on the parallel of latitude employed, the respective "lines of position" are drawn through these points, the spot at which they intersect giving the true place of the ship.

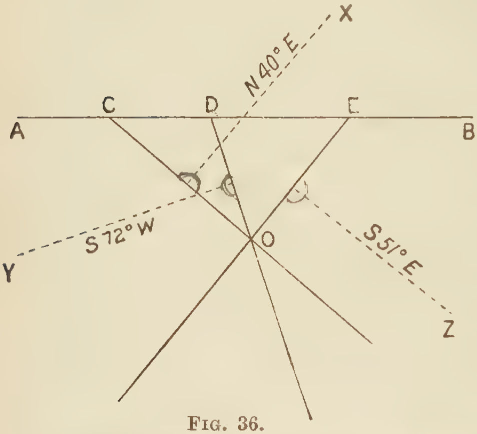


Fig. 36.

Thus in Fig. 36, supposed to be part of a Mercator's chart, let A B be part of the parallel of latitude used in finding the hour angles of the heavenly bodies X, Y, and Z, which give the longitudes respectively as C, D, and E, and whose true bearings or azimuths are N. 40° E., S. 72° W., and S. 51° E. The lines drawn on the charts at right angles to these bearings passing through C, D, and E, and intersecting at O are "lines of position," and O is the true place of the ship.

EXAMPLES.

(1) On November 18th, 1895, at about 9 p.m., in latitude D.R. 51° 17′ N., and longitude D.R. 6° 36′ W., when a chronometer showed 9h. 50m. 10s., whose error on G.M.T. was 22m. 36s. fast, the observed altitude of Capella (east of meridian), was 48° 20′ 20″, and at the same moment that of Markab (west of meridian) was 47° 8′ 30″, height of the eye, 10ft. Required, the ship's position.

Ship's Time	9h.	0m.	Nov. 18.	
Long. in T.		26m.		
Greenwich Time	9h.	26m.	,,	

Chronometer showed	9h.	50m.	10s.
Error fast …………		22m.	36s.
G.M.T., Nov. 18……	9h.	27m.	34s.

	Right Ascen. Mean Sun.		
	15h.	49m.	6·9s.
Accel. for 9h.		1m.	28·7s.
,, 27m.			4·4s.
,, 34s.			·1s.
At sights…	15h.	50m.	40·1s.

Capella (East of Meridian).

Obs'd alt. ………………	48°	20′	20″
Dip for 10ft. ……………		— 3′	7″
Apparent Altitude ……	48°	17′	13″
Refraction ………………		—	52″
True Altitude …………	48°	16′	21″
	90°	0′	0″
Zenith Distance…………	41°	43′	39″

Markab (West of Meridian).

Obs'd alt. ………………	47°	8′	30″
Dip for 10ft. ……………	—	3′	7″
Apparent Altitude ……	47°	5′	23″
Refraction ………………		—	54″
True Altitude…………	47°	4′	29″
	90°	0′	0″
Zenith Distance ………	42°	55′	31″

Lat.	51° 17′ N.		2038
Dec.	45° 53½′ N.		1574
Mer. Zen. Dist.	5° 23½′	45	
Zen. Dist.	41° 43½′	2536	
	2h. 45m. 18s.	2491	3962
			7574

H.A.	4h.	18m.	39s.	(East of Mer.)
	24h.	0m.	0s.	
,,	19h.	41m.	21s.	
R.A.	5h.	9m.	2s.	
Sidereal Time	24h.	50m.	23s.	
R.A.M.S.	15h.	50m.	40s.	
S.M.T.	8h.	59m.	43s.	
G.M.T.	9h.	27m.	34s.	
Long. in Time	0h.	27m.	51s.	

Longitude .... 6° 57′ 45″ W.

Lat.	51° 17′ N.		2038
Dec.	14° 39′ N.		144
Mer. Zen. Dist.	36° 38′	1975	
Zen. Dist.	42° 55½′	2677	
	1h. 26m. 24s.	702	8465
			0647

H.A.	1h.	51m.	29s.	(West of Mer.)
R.A.	22h.	59m.	35s.	
Sidereal Time	24h.	51m.	4s.	
R.A.M.S.	15h.	50m.	40s.	
S.M.T.	9h.	0m.	24s.	
G.M.T.	9h.	27m.	34s.	
Long. in Time	0h.	27m.	10s.	

Longitude .... 6° 47′ 30″ W.

The declination of Capella being more than 23½°, Burdwood's Tables are of no use. The true bearing can be obtained either from Blackburne's Tables; or, by inspection, from Johnson's bearings of the principal fixed stars.

BY BLACKBURNE'S TABLES.

Capella (East of Meridian).

H.A. and latitude + ·593
" declination — 1·143
Diff. long. corrn. — ·550
Corresponding departure for latitude 51° — ·346

With this departure in Table II. we find the true azimuth to be:

N. 70° 54′ E.

giving lines of position as

S. 19° 6′ E. and N. 19° 6′ W.

Markab (West of Meridian).

H.A. and latitude + 2·333
" declination — ·560
Diff. long. corrn. + 1·773
Corresponding departure for latitude 51° — 1·114

With this departure in Table II. we find the true azimuth to be:

S. 41° 54′ W.

giving lines of position as

N. 48° 6′ W. and S. 48° 6′ E.

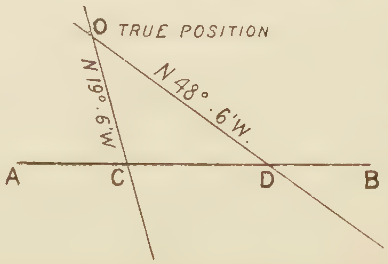
These drawn on the chart, Plate IV., give the true position of the ship as 51° 25′ 20″ N., and 7° 2′ 20″ W.

But by Blackburne's Tables we can find the true latitude and longitude of the ship without the chart in the following way:

To Find the Latitude.

Longitude by Capella ……	6° 57′ 45″ W.	— ·550		
„ „ Markab……	6° 47′ 30″ W.	+ 1·773	Latitude D.R………	51° 17′ 0″ N.
Difference in longitude……	10′ 15″ divided by 1·223 =		Corrn. in latitude …	8′ 23″ N.
			True latitude ………	51° 25′ 23″ N.

The diff. lat. correction is applied to the northward, because, as the following rough figure shows, the lines of position can only intersect to the northward of the erroneous parallel of latitude used. Let A B be part of the parallel of latitude 51° 17′ N., and C and D the longitudes obtained by the altitudes of Capella and Markab respectively. It is evident that the lines of position drawn through C and D can only meet at O to the northward of the latitude used; and also that the true position of the ship O lies to the westward of both positions obtained with the erroneous latitude. It is always best to draw a figure like this, then no mistake can possibly be made in applying the corrections.



TO FIND THE LONGITUDES.

Longitude by Capella...............	6°	57′	45″	W.
Diff. lat. correction 8·38 × ·55 ...		4′	36″	W.
True longitude........................	7°	2′	21″	W.

Longitude by Markab..............	6°	47′	30″	W.
Diff. lat. correction 8·38 × 1·773...		14′	51″	W.
True longitude........................	7°	2′	21″	W.

So we have Blackburne's Tables :

Latitude......... 51° 25′ 23″ N. Longitude......... 7° 2′ 21″ W.

which agrees with the position laid down on the chart, Plate IV., No. I.

(2) On August 18th, 1895, at 7h. 25m. p.m., in latitude D.R. 49° 32′ N., and longitude D.R. 4° 44′ W., when a chronometer showed 8h. 10m. 22s., whose error on G.M.T. was 20m. 43s. fast, the observed altitude of Arcturus (west of meridian) was 42° 38′ 10″, and at the same moment the observed altitude of Vega (east of meridian) was 72° 55′ 20″, the Index error in each case being—0′ 50″, height of the eye 22 feet. Required the true position of the ship.

Ship Time	7h.	25m.
Long. in Time ...		19m.
Greenwich Time...	7h.	44m.

Chronometer showed ...	8h.	10m.	22s.
Error Fast		20m.	43s.
G.M.T. Aug. 18th......	7h.	49m.	39s.

	R. A.	M.	S.
	9h.	46m.	23·9s.
Accel. for 7h.		1m.	9·0s.
,, 49m.			8·0s.
,, 39s.			·1s.
	9h.	47m.	41·0s.

Arcturus (West of Meridian).

Obs'd Altitude	42°	38′	10″
Index Error	—	0′	50″
	42°	37′	20″
Dip. for 22 feet...........	—	4′	37″
Apparent Altitude	42°	32′	43″
Refraction	—	1′	3″
True Altitude	42°	31′	40″
	90°	0′	0″
Zenith Distance.........	47°	28′	20″

Lat..........	49°	32′ N		....	1877
Dec.	19°	43½′ N		....	262
Mer. Z.D..	29°	48½′	... 1323		
Z.D.	47°	28½′	... 3241		
2h. 24m. 18s.			1918	...	2827
					4966

H.A.	3h.	6m.	40s.	(W. of Mer.)
R.A.	14h.	10m.	54s.	
Sidereal Time	17h.	17m.	34s.	
R.A.M.S.	9h.	47m.	41s.	
S.M.T.	7h.	29m.	53s.	
G.M.T...	7h.	49m.	39s.	
Long. in Time......	0h.	19m.	46s.	
Longitude	4°	56′	30″ W.	

Vega (East of Meridian).

Obs'd Altitude	72°	55′	20″
Index Error	—	0′	50″
	72°	54′	30″
Dip for 22 feet	—	4′	37″
Apparent Altitude.........	72°	49′	53″
Refraction		—	19″
True Altitude	72°	49′	34″
	90°	0′	0″
Zenith Distance	17°	10′	26″

Lat..........	49°	32′ N		....	1877
Dec.........	38°	41′ N		....	1076
Mer. Z.D.	10°	51′	.. 178		
Z.D.	17°	10½′	... 446		
0h. 53m. 10s.			268	...	4279
					7232

H.A.	1h.	14m.	52s.	(E. of Mer.)
	24h.	0m.	0s.	
H.A.	22h.	45m.	8s.	(W. of Mer.)
R.A.	18h.	33m.	26s.	
Sidereal Time	17h.	18m.	34s.	
R.A.M.S.	9h.	47m.	41s.	
S.M T.	7h.	30m.	53s.	
G.M.T.	7h.	49m.	39s.	
Long. in Time	0h.	18m.	46s.	
Longitude	4°	41′	30″ W.	

BY BLACKBURNE'S TABLES.

H.A. and latitude	+ 1·105
,, ,, declination	− ·496
Diff. long. correction	+ ·609
Corresponding departure (lat. 49½°)	·397

True bearing S. 68° 21′ W.

Line of position N. 21° 39′ W. and S. 21° 39′ E.

H.A. and latitude	+ 3·45
,, ,, declination	− 2·49
Diff. long. correction	+ 0·96
Corresponding departure (lat. 49½)	·623

True bearing S. 58° 4′ E.

Line of position N. 31° 56′ E. and S. 31° 56′ W.

Longitude by Arcturus (W.)	4° 56′ 30″ W.	+ ·609
,, ,, Vega (E.)	4° 41′ 30″ W.	+ ·96
	15′ 0″ divided by	1·569 = Correction

Latitude D.R.	49° 32′ N.
Correction	9′ 34″ S.
True latitude	49° 22′ 26″ N.

Long. by Arcturus	4° 56′ 30″ W.
9·56 × ·609	5′ 49″ E.
True longitude	4° 50′ 41″ W.

Long. by Vega	4° 41′ 30″ W.
9·56 × ·96	9′ 11″ W.
True longitude	4° 50′ 41″ W.

which agrees with the position laid down in the chart, Plate IV., No. II.

(3) On May 28th, 1895, at about 2 a.m., in latitude D.R. 49° 45′ 30″ N., and longitude D.R. 3° 56′ W., when a chronometer showed 3h. 7m. 45s., whose error on G.M.T. was

54m. 53s. fast, the Index error being *nil*, and the height of the eye 11 feet, the following altitudes were observed simultaneously :

Markab (East of Meridian)	24° 10′ 0″
Altair (East of Meridian)........	45° 23′ 40″
Arcturus (West of Meridian)	32° 51′ 30″

Required, the true position of the ship.

Ship Time, May 27th	14h. 0m.
Long. in Time	16m.
Greenwich Time ...	14h. 16m.

Chronometer showed	3h. 7m. 45s.
Error fast	54m. 53s.
G.M.T., May 27th ...	14h. 12m. 52s.

R.A.M.S. 27th	4h. 19m. 9·59s.
Accn. for 14h.	2m. 18·00s.
12m.	1·97s.
52s.	·14s.
At sights ...	4h. 21m. 29·7s.

Markab (E.)

Obs'd Alt..........	24° 10′ 0″
Dip. 11ft.	3′ 16″
	24° 6′ 44″
Refn.	2′ 9″
Tr. Alt............	24° 4′ 35″
	90° 0′ 0″
Zen. Dist.........	65° 55′ 25″

Altair (E.)

Obs'd Alt.	45° 23′ 40″
Dip. 11ft.	3′ 16″
	45° 20′ 24″
Refn...............	57″
Tr. Alt............	45° 19′ 27″
	90° 0′ 0″
Zen. Dist.........	44° 40′ 33″

Arcturus (W.)

Obs'd Alt.........	32° 51′ 30″
Dip. 11ft.........	3′ 16″
	32° 48′ 14″
Refn...............	1′ 30″
Tr. Alt............	32° 46′ 44″
	90° 0′ 0″
Zen. Dist.........	57° 13′ 16″

Lat. ...... 49° 45½′ N. 1897
Dec. ...... 14° 38½′ N. 144
M.Z.D. .... 35° 7′ ... 1820
Z.D. ...... 65° 55½′ ... 5921
3h. 35m. 24s. 4101 6129
8170

H.A. ...... 4h. 39m. 32s. (E.)
24h.
H.A. ...... 19h. 20m. 28s. (W).
R.A. ...... 22h. 59m. 33s.
42h. 20m. 1s.
R.A.M.S. 4h. 21m. 30s.
S.M.T. ...... 13h. 58m. 31s.
G.M.T. ... 14h. 12m. 52s.
Long. in T. 0h. 14m. 21s.

Longitude...... 3° 35′ 15″ W.

Lat. ... 49° 45½′ N. 1897
Dec. ... 8° 35½′ N. 49
M.Z.D. 41° 10′ ... 2472
Z.D. ... 44° 40½′ ... 2889
1h. 6m. 24s. 417 6200
8146

H.A. ...... 1h. 23m. 16s. (E.)
24h.
H.A. ...... 22h. 36m. 44s. (W.)
19h. 45m. 42s.
42h. 22m. 26s.
R.A.M.S. 4h. 21m. 30s.
S.M.T. ... 14h. 0m. 56s.
G.M.T. ... 14h. 12m. 52s.
Long. in T. 0h. 11m. 56s.

Longitude...... 2° 59′ W.

Lat. ...... 49° 45½′ N. 1897
Dec. ... 19° 43½′ N. 262
M.Z.D. .... 30° 2′ ... 1343
Z.D. ...... 57° 13¼′ ... 4587
3h. 10m. 0s. 3244 5111
7270

H.A. ...... 4h. 8m. 44s. (W.)
R.A. ...... 14h. 10m. 55s.
18h. 19m. 39s.
R.A.M.S. 4h. 21m. 30s.
S.M.T. ... 13h. 58m. 9s.
G.M.T. ... 14h. 12m. 52s.
Long. in T. 0h. 14m. 43s.

Longitude...... 3° 40′ 45″ W.

True bearing Markab (Burdwood)... N. 96° E.
" ... S. 84° E.
Line of position.............. N. 6° E. and S. 6° W.

True bearing Altair .... N. 150° E.
" .... S. 30° E.
Line of position...... N. 60° E. and S. 60° W.

True bearing Arcturus......... N. 98½° W.
" .... S. 81½° W.
Line of position .... N. 8½° W. and S. 8½° E.

BY BLACKBURNE'S TABLES.

H.A. and Latitude	+ ·430	H.A. and Latitude	+ 3·106	H.A. and Latitude	+ ·625
,, Declination	− ·278	,, Declination	− ·429	,, Declination	− ·407
	+ ·152		+ 2·677		·218

TRUE POSITION BY MARKAB AND ALTAIR.

Longitude Markab (E.)	3° 35′ 15″ W.	+ ·152			
,, Altair (E.)	2° 59′ 0″ W.	+ 2·677		Latitude D.R.	49° 45′ 30″ N.
	0° 36′ 15″	÷ 2·525	=	Correction	14′ 21″ S.
				True Latitude	49° 31′ 9″ N.
Longitude Markab	3° 35′ 15″ W.			Longitude Altair	2° 59′ 0″ W.
14′·35 × .152	2′ 11″ W.			14′·35 × 2·677	38′ 25″ W.
True Longitude	3° 37′ 26″ (W.)			True Longitude	3° 37′ 25″ W.

TRUE POSITION BY MARKAB AND ARCTURUS.

Longitude Markab (E.)	3° 35′ 15″ W.	+ ·152			
,, Arcturus (W.)	3° 40′ 45″ W.	+ ·218		Latitude D.R.	49° 45′ 30″ N.
	5′ 30″	÷ ·370	=	Correction	14′ 52″ S.
				True Latitude	49° 30′ 38″ N.
Longitude Markab	3° 35′ 15″ W.			Longitude Arcturus	3° 40′ 45″ W.
14′·86 × ·152	2′ 15″ W.			14′·86 × ·218	3′ 14″ E.
True Longiture	3° 37′ 30″ W.			True Longitude	3° 37′ 31″ W.

TRUE POSITION BY ALTAIR AND ARCTURUS.

Longitude Altair (E.)	2° 59′ 0″ W.	+ 2·677	Latitude D.R.	49° 45′ 30″ N.
„ Arcturus (W.)	3° 40′ 45″ W.	+ ·218		
	0° 41′ 45″	÷ 2·895	= Correction	14′ 25″ S.
			True Latitude	49° 31′ 5″ N. ✓

Longitude Altair	2° 59′ 0″ W.	Longitude Arcturus	3° 40′ 45″ W.
14′·42 × 2·677	38′ 36″ W.	14′·42 × ·218	3′ 9″ E.
True Longitude	3° 37′ 36″ W.	True Longitude	3° 37′ 36″ W. ✓

See Plate IV., No. III.

CHAPTER XXII.

THE NEW NAVIGATION OR DOUBLE CHRONOMETER PROBLEM No. 2.

IN the last chapter we have considered the double chronometer problem when simultaneous altitudes of two and three heavenly bodies have been observed. We will now deal with observations taken at intervals of time during which the ship has altered her position. In other words, in which the run of the ship must be taken into account.

Before laying down lines of position on the chart it is necessary to reduce all observations to a common parallel of latitude. Supposing an altitude of the sun is taken at 9.30 a.m. when using an erroneous latitude, say 49° 50′ N, the longitude is found to be 17° 20′ W. And again at 2.30 p.m., after the ship has run N. 27° E. (true) 28′, another altitude of the sun is taken. The run of the ship must be applied to the position found by the first observation, and the latitude thus obtained used for finding the hour angle, and thence the longitude by the second observation. If the longitudes thus found by the first and second observations agree, the latitude is correct. If they differ, lines of position must be laid through the position as shown in the preceding chapter.

4

EXAMPLES.

(1) On December 12th, 1895, at about 10 a.m., in Lat. D.R. 48° 35′ N. and Long. D.R. 7° 10′ W., when a chronometer showed 10h. 15m. 5s., whose error on G.M.T. was 9m. 54s., the observed altitude of the sun's lower limb was 13° 47′ 30″, height of eye 17ft. The ship then ran N.E. (true) 25 knots, and at about 2.10 p.m., when the same chronometer showed 2h. 20h. 52s., the observed altitude of the sun's lower limb was 12° 24′ 30″. Required the true position of the ship at the time of the second observation.

1st Observation.

Ship Time on the 11th	22h. 0m. 0s.
Long. in Time	28m. 40s. W.
Greenwich Time on the 11th. ...	22h. 28m. 40s.
Chronometer showed	10m. 15h. 5s.
Error slow	9m. 54s.
G.M.T. on the 11th................	22h. 24m. 59s.
Observed Altitude	13° 47′ 30″
Correction 17ft.	8′ 18″
True Altitude	13° 55′ 48″
	90° 0′ 0″
Zenith Distance	76° 4′ 12″

Declination on Dec. 12...........	23° 5′ 48″·8 S.
11″·19 × 1·6	17″·9
Declination at Sights	23° 5′ 30″·9 S.
Equation of Time on Dec. 12th	6m. 8·95s.
1·175s. × 1.6	1·88s.
Equation of Time at Sights	—6m. 10·83s.

Lat.	48° 35′ N.		1794
Declination ...	23° 5½′ S..................		363
M.Z.D.	71° 40½′	6856	
Z.D.	76° 4′	7592	
	1h. 28m. 32s. ...	736 ...	8676
			0833

Sun's H.A.	1h.	54m.	0s. (E. of Meridian)
	24h.	0m.	0s.
11th S.A.T.	22h.	6m.	0s.
E.T.		− 6m.	11s.
11th S.M.T.	21h.	59m.	49s.
„ G.M.T.	22h.	24m.	59s.
Long. in Time	0h.	25m.	10s.
Longitude	6°	17″	30′ W.

Blackburne's Tables.

H.A. and Latitude	+ 2·07
„ „ Declination	+ ·89
Diff. Long. correction	+ 2·96

Burdwood's Tables.

True Bearing	N. 153° E.
„	S. 27° E.
Line of Position	N. 63° E. and S. 63° W.

Run of Ship.

N. 45° E. 25 knots = Diff. Lat. 17·7 N. and Dep. 17·7 E.

Lat. D.R. 1st Observn.	48° 35′ N.
Diff. Lat.	17′ 42″ N.
Lat. D.R. 2nd Observn.	48° 52′ 42″ N.

Long. 1st Observn.	6° 17′ 30″ W.
Diff. Long.	26′ 30″ E.
Long. in.	5° 51′ 0″ W.

2nd Observation.

Ship Time on the 12th	2h.	10m.		Declination on Dec. 12	23°	5′	48″·8
Long. in Time		23m.	24s.	11″·19 × 2·5			27″·9
Greenwich Time on the 12th	2h.	33m.	24s.	Declination at sights	23°	6′	16″·7 S.
Chronometer showed	2h.	20m.	52s.	Equation of Time on Dec. 12		6m.	8·95s.
Error slow		9m.	54s.	1·175s. × 2·5			2·94s.
G.M.T. on the 12th	2h.	30m.	46s.	Equation of Time at sights		—6m.	6·01s.

Observed Altitude	12°	24′	30″	Lat.	48°	52½′ N.		1820
Correction 17ft		8′	0″	Dec.	23°	6½′ S.		364
True Altitude	12°	32′	30″	M.Z.D.	71°	59′	6907	
	90°	0′	0″	Z.D.	77°	27½′	7828	
Zenith Distance	77°	27′	30″	1h. 39m. 8s.			921	9643
								1827

S.A.T. on the 12th	2h.	8m.	9s.
Eq. Time		— 6m.	6s.
S.M.T. on the 12th	2h.	2m.	3s.
G.M.T. on the 12th	2h.	30m.	46s.
Long. in Time	0h.	28m.	43s.

Longitude .... 7° 10′ 45″ W.

Blackburne's Tables.

H.A. and Latitude	+ 1·840
„ „ Declination	+ ·801
Diff. Long. Correction	+ 2·641

Burdwood's Tables.

True Bearing		N. 150° W.
	or	S. 30° W.
Line of position		N. 60° W. and S. 60° E.

By Blackburne's Tables.

By 1st Observation Longitude at 2.10 p.m.........	5° 51′ W.	+ 2·96		Lat. D.R. at 2 p.m.
„ 2nd „ „ „ „	7° 10′ 45″ W.	+ 2·641		48° 52′ 42″ N.
	1° 19′ 45″ divided by	5·601	=	14′ 14″ S.
		Correct Latitude......		48° 38′ 28″ N.

1st Observation.

Longitude 2.10 p.m.	5° 51′ W.
14·24 × 2·96	42′ 9″ W.
True Longitude	6° 33′ 9″ W.

2nd Observation.

Longitude 2.10 p.m.........	7° 10′ 45″ W.
14·24 × 2·641	37′ 36″ E.
	6° 33′ 9″ W.

On Plate IV., No. IV., the lines of position laid down give the true place of the ship as above.

(2) On August 16th, 1895, at 5.30 p.m., in Lat. D.R. 50° 27′ N. and Long. D.R. 5° 57′ W., the observed alt. of the sun's lower limb was 16° 45′ 20″, Index Error — 1′ 10″, height of

eye 15ft. when a chronometer showed 4h. 38m. 2s., whose error on G.M.T. was 1h. 10m. 12s. slow.

The ship then ran N. 13° E. (true) 27 miles, and at 8.10 p.m., when the same chronometer showed 7h. 27m. 27s., the observed alt. of Antares (West of Meridian) was 10° 31′ 0″; Index Error and height of eye as above.

Required the true position of the ship.

Ship Time, 16th	5h. 30m.	Chronometer showed	4h. 38m. 2s.	Declination	13° 45′ 7″·1 N.
Long. in Time	23·8	Error slow	1h. 10m. 12s.	47·42 × 5·8	4′ 35″
Greenwich Time, 16th	5h. 53·8m.	G.M.T. on 16th	5h. 48m. 14s.	At Sights	13° 40′ 32″ N.
				Equat. of Time	+ 4m. 8s·6
				·502 × 5·8	2s·9
				At Sights	+ 4m. 5s·7

Observed Alt.	16° 45′ 20″	Lat.	50° 27′ N.		1961
I.E.	− 1′ 10″	Dec.	13° 40′½ N.		125
	16° 44′ 10″	M.Z.D.	36° 46′½	1989	
Correction 15ft.	+ 9′ 0″	Z.D.	73° 7′	7096	
True Alt.	16° 53′ 10″		4h. 2m. 48s.	5107	7081
	90° 0′ 0″				9167
Zen. Dist.	73° 6′ 50″				

S.A.T.	5h. 19m. 48s.
Eq. Time	+ 4m. 6s.
S.M.T.	5h. 23m. 54s.
G.M.T.	5h. 48m. 14s.
Long. in Time	0h. 24m. 20s.
Longitude	6° 5′ 0″ W.

RUN OF SHIP.

N. 13° E. (true) 27 miles.

Diff. Lat.	26·3	Departure	6·1

Lat. D.R	50° 27′ N.	Long. Obs.	6° 5′ W.
Diff. Lat.	26′ N.	Diff. Long	10′ E.
8.10 p.m. Lat. D.R.	50° 53′ N.	Long.	5° 55′ W.

ANTARES (WEST OF MERIDIAN).

Chronometer showed	7h. 27m. 27s.	R.A.M.S.	9h. 38m. 30·77s.
Error slow	1h. 10m. 12s.	Acceleration for 8 hours	1m. 18·85s.
G.M.T. on 16th	8h. 37m. 39s.	,, ,, 37m.	6·08s.
		,, ,, 39s.	·11s.
			9h. 39m. 55·81s.

Observed Alt.	10° 31′ 0″
I.E.	— 1′ 10″
	10° 29′ 50″
Dip for 15ft.	— 3′ 49″
	10° 26′ 1″
Refraction	— 5′ 7″
True Alt.	10° 20′ 54″
	90° 0′ 0″
Zen. Dist.	79° 39′ 6″

Lat.	50° 53′ N.		2001
Dec.	26° 12′ S.		471
M.Z.D.	77° 5′	7764	
Z.D.	79° 39′	8204	
	1h. 8m. 16s.	440	6439
			8911

Antares H.A.	1h. 31m. 0s. (W. of Meridian)
„ R.A.	16h. 23m. 1s.
Sidereal Time	17h. 54m. 1s.
R.A.M.S.	9h. 39m. 56s.
S.M.T.	8h. 14m. 5s.
G.M.T.	8h. 37m. 39s.
Long. in Time	0h. 23m. 34s.
Longitude	5° 53′ 30″ W.

To Find True Position.

Sun (by Burdwood's Tables).		Antares (by Johnson's Bearings of Principal Stars).	
True Bearing	N. 89° W.	True Bearing	S. 20° W.
Line of position	S. 1° W. and N. 1° E.	Line of position	N. 70° W. and S. 70° E.

which give the position of the ship as latitude 50° 53′ 40″ N., and longitude 5° 55′ W. (Vide Plate IV., No. V.)

In the preceding problem we are unable to use either Burdwood's or Blackburne's Tables to find the true bearing of Antares, as his declination exceeds 23½°, which renders Burdwood useless, and Blackburne only gives the necessary elements for four stars exceeding this declination, viz., Vega, Capella, Crucis, and Polaris. The bearing must be found by the ordinary altitude azimuth problem, or it is given in Johnson's "Bearings of the Principal Stars," which I have employed in this instance.

(3) On September 15th, 1895, at about 4 a.m., in latitude D.R., 49° 30′ N., and longitude D.R. 7° 0′ W., when a chronometer showed 3h. 25m. 5s., whose error on G.M.T. was 1h. 5m. 45s. slow, the following simultaneous altitudes were observed :

Moon (East of Meridian)	29° 56′ 40″ (lower limb)
Rigel (East of Meridian)	29° 3′ 20″

the index error being *nil*, and the height of the eye 21ft.

The ship then ran N. 66½° E. (true) 23 miles, when at 8 a.m. the observed altitude of the sun's lower limb was 22° 9′ 20″, and the same chronometer showed 7h. 21m. 57s. Index error and dip as above. Required, the ship's position at 8 a.m.

1st Observations at 4 a.m.

Ship Time on Sept. 14	16h.	0m.	
Long. in Time		28m.	
Greenwich Time on Sept. 14	16h.	28m.	
Chronometer showed	3h.	25m.	5s.
Error slow	1h.	5m.	45s.
G.M.T. on Sept. 14	16h.	30m.	50s. ✓

R.A.M.S.

On Sept. 14	11h.	32m.	50·9s.
Acceleration for 16h.		2m.	37·7s.
,, ,, 30m.			4·9s.
,, ,, 50s.			·1s.
At sights	11h.	35m.	33·6s.

Moon (East of Meridian).

Right Ascension at 16h.	8h.	13m.	16s.
Variation in 31m.		1m.	18s.
Right Ascension at sights	8h.	14m.	34s. ✓
Declination at 16h.	23°	36′	8″ N.
Variation in 31m.		5′	19″
Declination at sights	23°	30′	49″ N.

Rigel (East of Meridian).

Right Ascension	5h.	9m.	32s. ✓
Declination	8°	19′	0″ S. ✓

Hor. Parallax.

At Midnight	59′ 44″·3
,, Noon	60′ 7″·6
,, 4.30 a.m.	59′ 53″

Semidiameter.

At Midnight	16′ 18″·3
,, Noon	16′ 24″·7
,, 4.30 a.m.	16′ 21″
Augmentation.	9″
At sights	16′ 30″

Observed Alt.	29°	56′	40″	
Dip for 21ft.		4′	31″	
	29°	52′	9″	
Semid.		16′	30″	
App. Alt.	30°	08′	39″	
Corr. in Alt.		49′	22″	
			45″	
True Alt.	30°	58′	46″	
	90°	0′	0″	
Zen. Dist.	59°	1′	14″	

Lat.	49°	30′ N.			1874
Dec.	23°	31′ N.			377
M.Z.D.	25°	59′	1011		
Z.D.	59°	1′	4852		
3h. 27m. 56s.			3841	...	5844
					8095

H.A.	4h.	36m.	48s.	(E.)
H.A.	19h.	23m.	12s.	(W.)
R.A.	8h.	14m.	34s.	
Sid. Time	27h.	37m.	46s.	
R.A.M.S.	11h.	35m.	34s.	
S.M.T.	16h.	2m.	12s.	
G.M.T.	16h.	30m.	50s.	
Long. in Time	0h.	28m.	38s.	
Longitude	7°	9′	30″ W.	

Observed Alt.	29°	3′	20″
Dip for 21ft.		— 4′	31″
App. Alt.	28°	58′	49″
Refraction		— 1′	45″
True Alt.	28°	57′	4″
	90°	0′	0″
Zen. Dist.	61°	2′	56″

Lat.	49°	30′ N.			1874
Dec.	8°	19′ S.			46
M.Z.D.	57°	49′	4674		
Z.D.	61°	3′	5160		
1h. 11m. 44s.			486	...	6860
					8780

H.A.	1h.	29m.	36s.	(E.)
H.A.	22h.	30m.	24s.	(W.)
R.A.	5h.	9m.	32s.	
Sid. Time	27h.	39m.	56s.	
R.A.M.S.	11h.	35m.	34s.	
S.M.T.	16h.	4m.	22s.	
G.M.T.	16h.	30m.	50s.	
Long. in Time	0h.	26m.	28s.	
Longitude	= 6°	37′ W.		

By Blackburne's Tables.

H.A. and Lat.	+ ·450	H.A. and Lat.	+ 2·830
H.A. and Dec.	− ·465	H.A. and Dec.	+ ·382
	− ·015		+ 3·212

Longitude, Moon (E. of Mer.)	7° 9′ 30″ W.	− ·015		
,, Rigel (E. of Mer.)	6° 37′ 0″ W.	+ 3·212	Lat. D.R.	49° 30′ N.
	0° 32′ 30″ divided by 3·197		Corrn.	10′ 10″ S.
			True Lat.	49° 19′ 50″ N.

Longitude, Moon	7° 9′ 30″ W.	Longitude, Rigel	6° 37′ W.
10·16 × ·015	9″ W.	10·16 × 3·212	32′ 38″ W.
True Longitude	7° 9′ 39″ W.	True Longitude	7° 9′ 38″ W.

By Burdwood.

Moon.		Rigel.	
True bearing	East.	True bearing	N. 155° E. = S. 25° E.
Line of position	North and South.	Line of position	N. 65° E & S 65° W.

Run of Ship.

Diff. Lat. …… 9·3 N. Departure ……… 21·1 E. N. 66½° E. …… 23·0

Lat. left	49° 19′ 50″ N.	Long. left	7° 9′ 38″ W.
Diff. lat.	9′ 18″ N.	Diff. long	32′ 30″ E.
Lat. in.	49° 29′ 8″ N.	Long. in.	6° 37′ 8″ W.

2nd Observation, 8 a.m.

Ship Time	8h. 0m. a.m. 15th.	Declination 15th Sept......	30° 1′ 40″·6 N.
Long. Time	26½m.	57″·81 × 3·5	3′ 22″·3
Greenwich Time ...	20h. 26½m. 14th	At Sights	3° 5′ 3″ N.
Chron. showed......	7h. 21m. 57s.	Equat. of Time	4m. 47·95s.
Error slow...........	1h. 5m. 45s.	,, 88s. × 3·5	3·08s.
G.M.T., 14th........	20h. 27m. 42s.	At sights	— 4m. 44·87s.

Obsd. Alt.	22°	9′	20″	Lat.	49° 29′ N.	....			1873
Corrn. for 21ft.	+	9′	6″	Dec.	3° 5′ N.	....			6
True Alt.	22°	18′	26″	M.Z.D.	46° 24′	...	3103	...	
	90°	0′	0″	Z.D.	67° 42′	...	6206	...	
Z.D.	67°	41′	34″		3h. 5m. 34s.	...	3103	...	4917
									6796

H.A.	3h.	54m.	11s.	(E.)
S.A.T.	20h.	5m.	49s.	(W.)
Eq. Time		4m.	45s.	
S.M.T.	20h.	1m.	4s.	
G.M.T.	20h.	27m.	42s.	
Long. in Time	0h.	26m.	38s.	
Longitude	6°	39′	30″ W.	

True Bearing Burdwood... N. 113° E. = S. 67° E.
Line of position............... N. 23° E. & S. 23° W,

By Blackburne's Tables.

Sun	
H.A. and Latitude	+ ·716
,, Declination	− ·061
	+ ·655

Longitude by Moon	6°	37′	8″ W.	−	·015
,, Sun	6°	39′	30″ W.	+	·655
		2′	22″ divided by ·670		

Lat. D.R.	49°	29′	8″ N.
Corrn.		3′	31″ N.
True Latitude	49°	32′	39″ N.

Longitude by Moon	6°	37′	8″ W.
3·52 × ·015			3″
Long. in.	6°	37′	11″ W.

Longitude by Sun	6°	39′	30″ W.
3·52 × ·655		2′	18″ E.
Long. in.	6°	37′	12″ W.

See Plate IV., No. VI.

CHAPTER XXIII.

TIDES.

THE generally accepted theory of the cause of the tides is the joint action of the sun and moon owing to the law of gravitation. The whole mass of the earth is attracted by these two bodies.

Let us imagine the earth to be covered uniformly with water to a considerable depth over its whole surface. The moon (whose attractive power with regard to the earth is, to the sun's, as $2\frac{1}{4} : 1$) will cause the water on the side of the earth nearest to her to be raised above its ordinary level; on exactly the opposite side of the earth the water will be raised in a similar manner; and as a consequence of this increase in the volume of water at these points, it will be depressed at other portions of the earth's surface, and this most of all at those parts of it which are equidistant from the positions of greatest elevation. These elevations of the water are called the tidal waves, and are represented in the following figure, in which, it is needless to say, the height of the sea above its ordinary level is immensely exaggerated.

Let the circle A B represent the earth and the line T T_1 the direction of the pull of the moon's gravitation, the moon being supposed to be to the right of the figure. The water on the earth over B will be raised to T; that over A to T_1, while the water at N and N_1 will be depressed. Among the theories which have been propounded to account for the inferior tidal wave—that is the tidal wave on the opposite side of the earth to the moon—there is one which it may be well to mention. It is that the mass of the earth being nearer the moon than the water on its further side, and gravitation acting inversely as the cube of the distance, the earth is dragged away from this water, thereby causing it to bulge out from the surface of the earth.

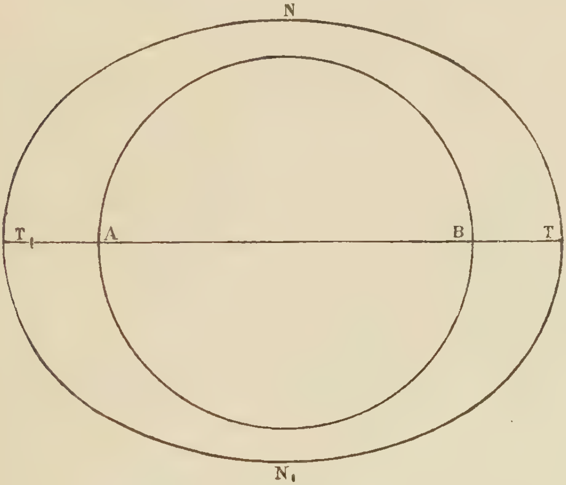


FIG. 37.

The moon appears to revolve round the earth in about 24hr. 48min., thereby causing two tidal waves to sweep round after her, so that we have two high tides and two low tides in that time.

Again, the attraction of the sun, though of much less importance in this regard, increases the elevation of these tidal waves when the sun and moon are in conjunction, or in opposition, that is when the sun, moon, and earth are nearly in one line, thus causing what are known as spring tides, but when they are in quadrature, that is when the sun and moon are at right angles to one another, we have neap tides.

These tidal waves are simply a rising and falling of the water; little or no horizontal stream is imparted to the water until either the decrease in its depth, or the shape of the coast, converts this perpendicular movement into a horizontal stream. To illustrate how this comes about, I would ask my reader to imagine himself to be standing on the perpendicular wall of a pier jutting out into the sea, with a considerable depth of water close to. There is no wind, and the sea is of a glassy smoothness; but there is a succession of rollers coming in from seaward. Look at the water alongside the pier, its level is continually changing; as the crest of each roller approaches the pier the water rises, to fall again as the hollow succeeds it. It is simply an up and down motion, and a representation in miniature of the effect of the tidal waves. Now let us go to the adjoining beach. What do we see? As each roller comes in, when it is near enough to have its momentum affected by the shoaling of the water, its summit gains horizontal velocity, and it eventually dashes against the shore at our feet, running some distance up the beach before it recedes. Similarly, the great tidal waves cause the tidal streams in narrow waters, or in positions where the configuration of the land creates resistance to their horizontal progression.

It must be clearly understood that the tidal waves are quite distinct from tidal streams; that high water and low water are terms applicable to the former alone, and flood and ebb to the latter. As a matter of common knowledge, it continually happens that in certain localities the water is falling before the flood-tide has ceased to make, and *vice versâ*. It must also be noted that, owing to the configuration of the coast line and other causes, the time of high water never coincides with the time of the moon's meridian passage, but occurs at various intervals of time thereafter.

There are therefore two divisions into which this subject naturally resolves itself; one is the alteration in the level of the sea, or the rise and fall of the tide; and the other tidal streams, or the horizontal motion of the water.

The tidal wave approaches our coasts from the westward. When the moon passes the meridian of Greenwich at about noon, or in other words, at full and change of the moon at Greenwich; then, at 4 p.m., the crest of the great tidal wave stretches from Brest on the coast of France to the north-westward, curving to the northward and running parallel to and at a short distance from the west coast of Ireland. It is evident that a line drawn along the crest of the tidal wave will be the line of high water. Therefore all places and coasts it touches will have high water at the same time; and this line is commonly called the co-tidal line.

At five p.m. this tidal wave has reached the south and west coasts of Ireland, and it curves up the Bristol Channel till within a few miles of St. Bride's Head, when it recurves south to the Scilly Islands and Land's End, and continues along the south coast of England as far as the Start Point, when, after bellying out a little to the eastward, it runs southerly to the coast of France.

The co-tidal line advances at the rate of from twenty to fifty miles an hour. It is divided by the coast of Ireland into two portions. The northern portion is again divided by the coast of Scotland into two parts, one proceeding to the northward and rounding the northern extremity of Scotland, the other finding its way through the Northern Channel into the Irish Sea. The southern portion passes the south coast of Ireland; part of it goes up the Irish Channel and eventually meets the tidal wave from the north in the centre of the Irish Sea, another part swells the water in the Bristol Channel, and the third part gives high water in succession to the ports in the English Channel. This latter portion at length meets in the Straits of Dover another tidal wave which has come round the north coast of Scotland and made its way to the southward along the east coasts of Scotland and England.

Let us for a moment glance at the position of affairs at 11 p.m. The tidal waves have met in the centre of the Irish Sea, and it is high water from Dublin to Carlingford Bay on the coast of Ireland, and from Liverpool Bay on the coast of England to the south of Scotland. It is also high water in the Straits of Dover and at the mouth of the Thames, where the English Channel and North Sea tidal waves have met. And another tidal wave causes it to be high water about the north-eastern extremity of Scotland.

At the same time it is low water on the south-west and south coasts of Ireland, on the south coast of England from the Scilly Islands and Land's End to the Start Point, and on the east coast of England about Flamborough Head.

The general result is that we have two tidal waves in the Irish Sea, one coming from the southward and the other from the northward; a tidal wave going up the Bristol Channel, another up the English Channel, and another sweeping round the northern extremity of Scotland and running down the east coasts of Scotland and England.

Let us now turn our attention to tidal streams.

In the Irish Channel when the northern tidal wave enters the Irish Sea about 7 P.M., that from the south comes in one hour earlier. The tidal streams which accompany these waves have no such difference in their times of flow and ebb in any part of the Irish Sea, but are found to commence and cease simultaneously. The current sets to the northward in the whole of its southern portion during the flow of the tide; and at the same time the current runs to the southward in its northern half during the same period. During the ebb of the tide exactly the reverse of this occurs. To the westward of the Isle of Man there exists an elliptical area between fifteen and twenty miles in diameter where no tidal streams exist, the currents there evidently neutralising each other.

The times of slack water in the Irish Sea correspond with the times of high and low water at Fleetwood in Morecambe Bay, which are twelve minutes earlier than at Liverpool. Thus we see that for nearly six hours after low water at Fleetwood the tidal streams pour into the Irish Sea from the northward and southward, meeting and neutralising each other in the neighbourhood of the Isle of Man; and then for nearly six hours after high water these streams are reversed, and run for the same period in opposite directions. Their velocity is about 2·5 knots in ordinary spring tides, except where the narrowing of the channel (as at the Mull of Cantyre, where the stream rushes along at the rate of five knots), the shoaling of the water, &c., cause the speed to vary.

In the English Channel and in the North Sea, as far as the parallel of 54° N. latitude, the time of the ebb and flood tides is regulated by the time of high water and low water at Dover. These streams run simultaneously throughout a considerable portion of the English Channel and North Sea. To the westward of a line joining the Bill of Portland and Cape La Hogue, however, the regularity of the tidal stream is interfered with by currents running into and out of the Gulf of St. Malo.

In the Bristol Channel, when the tide is rising at Fleetwood, a powerful stream will be found running out of it, and a south-westerly current exists on the north Cornish coast. When the tide is falling at that place there is a flow into this channel, and a north-easterly current on the north Cornish coast.

As regards the line of coast from the Scilly Islands to Ilfracombe, it is found that while the stream from Scilly is setting to the northward the stream from the Irish Channel is setting to the southward; these streams meet off the entrance of the Bristol Channel, and both turn into it. Thus, when the water is falling at Dover and at Fleetwood, in all the space to the eastward of a line joining Scilly and Tuskar the stream will be found setting to the eastward.

The tidal stream off the west coast of Scotland may be divided into two parts whose line of separation is the Isle of Skye.

With regard to that portion extending from the Mull of Cantyre to the Isle of Skye, the tidal stream is running to the northward from one hour before high water at Greenock to low water at Greenock, and to the southward from five hours before high water to about high water at that place.

To the northward of Skye the tide runs to the northward and to the eastward from three hours after high water to two hours before high water at Greenock; and *vice versâ* from two hours before high water to two hours after high water at that port.

Should my readers wish to pursue this most interesting subject, they cannot do better than read the manual of tides and tidal currents by the Rev. S. Haughton, M.D., F.R.S.

It is well to recollect that slack water lasts generally about three-quarters of an hour, and that the actual time during which the tidal streams run either way is about five hours and forty minutes, and that their greatest velocity is nearly always attained about three hours after slack water. As the speed gradually increases to its maximum and as gradually decreases, it is possible roughly to estimate it at any given time. It is advisable, however, to be provided with the Admiralty tidal charts to ensure accuracy.

CHAPTER XXIV.

TIDES (*continued*).

IN this chapter we will consider how the time of high water is found; how to correct soundings taken before comparing them with those given on the chart; and how to ascertain approximately the direction and rate of the tidal stream at any particular place.

The time of high water at all the more important places on the sea coast when the moon is full or new has been determined by observation. It is called the time of high water at full and change (H.W., F. & C.), or the establishment of the port, and is the interval between the time of the moon's meridian passage on the day of new or full moon (that is when the moon passes the meridian of the place at very nearly noon or midnight, and that of the high water which follows, expressed in hours and minutes. If the moon were the sole cause of the tides, high water would follow her meridian passage by about the same interval, and therefore would occur about 48 minutes later on each succeeding day. The disturbing influence of the sun's attraction, however, causes so much irregularity that it is impossible to lay down any general law for finding the time of high water.

In about eighteen years eleven days the sun and moon take up all the positions possible relative to each other and to the earth, and therefore it has been found that, by means of a series of observations made at certain important ports extending over that period, we are able to predict approximately the time of high water and its height. These ports are called the Standard Ports, and in the Admiralty Tide Tables these forecasts are tabulated for every day in the year. The height of tide given in these tables is the number of feet and inches it rises above the average height of low water at spring tides. The time given is the mean time at the place, but as on board ship apparent time is usually kept, it becomes necessary to apply the equation of time to the ship time in order to find the mean time with which to enter the tables. Also, when Greenwich or English railway time is kept on board, the longitude in time, which is given in the Admiralty Tide Tables at the bottom of the page under each Standard Port, must be applied.

This information is made available for other important places by tabulating the difference between the times and heights of the tide at such a place and those of the nearest Standard Port, called its port of reference. These differences are called tidal constants; they are given for the home ports in the Admiralty Tide Tables, 1895 (pp. 101 to 105).

The time of high water may be roughly computed by adding to the time of high water at full and change 48 minutes for each day of the moon's age.

At Penzance, H.W., F. & C., 3hrs. 30min. Find the time of high water on March 1st, 1895, P.M.?

Now, we find from the Nautical Almanac that the moon's age at noon on March 1st is 4 days 19hrs. 16min., or 4·8 days after change of the moon. We multiply 4·8 by 48 min., and add the result to the time of H.W., F. & C. :

H.W., F. & C., at Penzance	4hrs. 30min.
48min. × 4·8	3hrs. 50min.
Time of high water on March 1st......	8hrs. 20min. P.M.

But this is an exceedingly rough method, and should never be resorted to without necessity. Let us now, by means of the Admiralty Tide Tables, find the time of high water more accurately.

In the Admiralty Tide Tables for 1895 (page 104) we see that the port of reference or standard port for Penzance is Devonport, and that the constants are — 1h. 13m. and + 1ft. 0in. Turn to page 18; at Devonport on March 1st the P.M. high tide will take place at 8h. 24m., and we proceed thus :

March 1st.—H.W. at P.M. at Devonport...........	8h. 24m.
Constant for Penzance..................	— 1h. 13m.
H.W. at Penzance	7h. 11m. P.M.

a difference of no less than 1h. 9m. from the time computed by the moon's age.

It may be well to explain here that if the tidal constant is subtractive and greater than the time of H.W. given in the tables, 12 hours must be added to the latter, and the time found will be A.M. of the same day if the time given in the tables is P.M.; and P.M. of the day before, if the time given in the tables is A.M. Also that if the constant is additive, and together with the time given in the tables is more than 12 hours, then 12 hours must be deducted and the result will be P.M. time of the same date if the time given in the tables is A.M.; and A.M. time of the next day if the time in the tables is P.M.

(1) Thus on March 1st, 1895, on page 18, we see that the P.M. time high water at Portsmouth is 2h. 15m. Suppose we wish to find the time of H.W. at Portland Breakwater. At page 104 the constant for that place is—4h. 40m.:

March 1st.—H.W. at Portsmouth....................	2h. 15m. P.M.
Add..	12h. 0m.
	14h. 15m.
Constant, Portland Breakwater ...—	4h. 40m.
March 1st.—H.W. at Portland Breakwater........	9h. 35m. A.M.

(2) Also that the H.W. at Portsmouth A.M. is 2h. 0m. on the same date. Here we have:

March 1st.—H.W. at Portsmouth....................	2h. 0m. A.M.
Add..	12h. 0m.
	14h. 0m.
Constant for Portland Breakwater (page 104)—	4h. 40m.
Feb. 28th.—H.W. at Portland Breakwater........	9h. 20m. P.M.

(3) Again, at Greenock, on April 9th, the time of H.W. A.M. is at 11h. 39m. (page 30). We wish to find the time of H.W. at Oban:

April 9th.—Time of H.W. at Greenock	11h. 39m. A.M.
Constant for Oban (page 103).........+	5h. 20m.
	16h. 59m.
Deduct	12h. 0m.
Time of H.W. at Oban	4h. 59m. P.M.

(4) At Greenock, on July 20th, 1895, the P.M. H.W.

will take place at 10h. 31m. (page 54). Find time of H.W. at Oban:

July 20th.—Time of H.W. at Greenock	10h. 31m. P.M.
Constant for Oban	+ 5h. 20m.
	15h. 51m.
Deduct	12h. 0m.
July 21st.—Time of H.W. at Oban	3h. 51m. A.M.

If it is required to find the time of H.W. at a place not given in the Table of Tidal Constants, a constant for such place may be found by taking out of the Admiralty Tide Tables the time of H.W., F. & C., for the place required and for the nearest standard port, the difference between these times, marked − if the time at the standard port is greater and + if it is less, is the constant required.

Example.

Required the times of H.W. at Teignmouth on August 3rd, 1895.

The nearest standard port is Devonport.

H.W., F. & C., Devonport (page 169)	5h. 43m.
H.W., F. & C., Teignmouth (page 169)	6h. 0m.
Constant	+0h. 17m.

	A.M.	P.M.
August 3rd (Devonport)	3h. 52m.	4h. 23m.
Constant	+0h. 17m.	+0h. 17m.
H.W., Teignmouth, August 3rd, 1895	4h. 9m.	4h. 40m.

To find the time of H.W. at a foreign port, we first ascertain the tidal constant as in the preceding example, and apply it in the same way; but a correction is necessary, on account of the difference of time during which the moon has altered her position as regards the sun and the earth. This is found by adding together the longitudes of the foreign port and standard port, if of the same name, or by taking their difference if not, marking the result west if both are west longitude, and east if both are east; or, if they are of different names, west if the west longitude be greater, and east if the contrary. With this longitude, and the difference of the moon's meridian passage (taken out of the Admiralty Tide Table, as in examples worked below, or from the Nautical Almanac, page 4 in each month), for the day and the day before if the longitude is east, but for the given day and the day after if the longitude is west, enter Table 23, Inman's, or Table XVI., Norie, and we have the correction to be added if longitude is west and to be subtracted if east to the time of H.W. already found.

EXAMPLE.

Find the time of high water A.M. and P.M. at Falkland Sound, north entrance, on November 1st, 1895.

Brest, H.W., F. & C. (page 178)	3h. 47m.
Falkland Sound, N. entrance, H.W., F. & C. (page 179)	6h. 45m.
Constant	+2h. 58m.

Long., Brest	4° 29′ 36″ W.	moon's mer. passage, Nov. 1 (page 82)...	11h. 8m.
" Falkland S.	60° 4′ 0″ W.	moon's mer. passage, Nov. 2 (page 82)...	11h. 54m.
Same name: Sum	64° 33′ 36″ W.	Difference	0h. 46m.

H.W., Brest (page 82) Nov. 1	A.M., 2h. 39m.	P.M., 2h. 54m.
Constant for Falkland Sound	+2h. 58m.	+2h. 58m.
	5h. 37m.	5h. 52m.
Correction for 64° longitude and difference of moon's meridian passage (Table 23, Inman; Table XVI., Norie)	+ 9m.	+ 9m.
Time of H.W. at Falkland Sound	A.M., 5h. 46m.	P.M., 6h. 1m.

CORRECTION OF SOUNDINGS.

The soundings given on the Admiralty charts is the depth of the sea at low water, ordinary spring tides. As the rise and fall of the tide in some places is very considerable, it is evidently necessary to have some means of finding what correction must be applied to any sounding taken before comparing it with that on the chart. The best and simplest method is that given in the Admiralty Tide Tables by the use of Table B at page 98. It will be well to explain the meaning of certain terms used in this connection.

In Fig. 38, let B be the bottom of the sea. At ordinary spring tides the sea level rises to A at high water and falls to C at low water. At ordinary neap tides it only rises to D and falls to E. The line F G, equi-distant from A and C and from D and E, is half tide, or the mean level of the sea. A C is called the spring rise, or mean spring range ; D C is the neap rise ; D E is the neap range ; and C H the half mean spring range. The depth, C B, is that which would be given in the Admiralty charts, but the sounding by the lead might be any depth between C B and A B.

The rule for finding the correction to be applied to soundings taken, in order to compare them with those marked on the chart, is as follows :

Find the time of the high water which has taken place at the shortest interval of time from that of the sounding at the nearest port, and thence the interval between that time and the time of taking the sounding. Take out from the Admiralty Tide Tables the height of high water that tide and subtract from it the half mean spring range found in the Admiralty Tide Tables under the standard port, which will give us the height above the half tide or mean level of the sea at high water that tide. Then to the half mean spring range apply the correction taken out from Table B with the height above mean level of the sea at the side of the table and the interval of time from the time of high water at the top. This will give us the correction to be subtracted from the soundings taken, except in the very rare cases when the Table B correction is greater than the half mean spring range and is subtractive; then the cor-

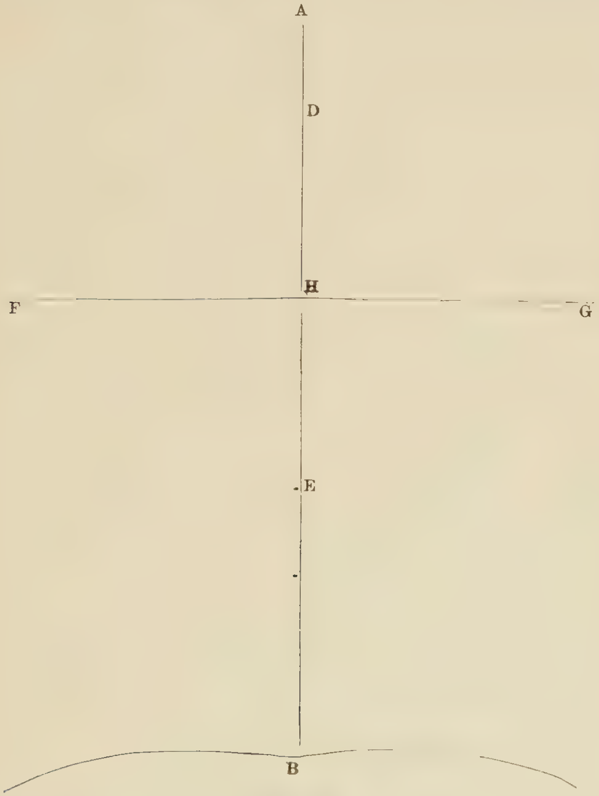


FIG. 38.

rection, which will be the amount by which the Table B correction exceeds the height of the mean level of the sea, must be added to the sounding taken.

1895. On March 18, at 3h. 16min. A.M., being off Brest by reckoning, took a cast of the lead. Required the correction to be applied to the depth obtained by the lead line before comparing it with the depth marked on the chart.

H.W. at Brest (page 18)	8h. 35min. a.m.	Height	13ft. 0in.
Time of sounding	3h. 16min. a.m.	Half mean spring range	9ft. 6in.
Time from H.W.=5h. 19min.		Height above mean sea level	3ft. 6in.

Half mean spring range .... 9ft. 6in.
Correction Table B, entered with 5h. 19min. and 3ft. 6in. (page 98)..—2ft. 11in.
Correction to be subtracted from sounding taken .... 6ft. 7in.

The correction from Table B is in this example averaged for the time and height required; but it is evident that in practice the nearest hour or half hour and the nearest height would be quite accurate enough. If we get within a fathom of the truth in deep sea sounding we ought to be well satisfied.

1895. March 12, off Liverpool by reckoning, took a sounding at 6.10 P.M., required correction to be applied before comparing with the chart.

H.W. at Liverpool (page 22)	0h. 2min. p.m.	Height of tide............	30ft. 0in.
Time of sounding	6h. 10min. p.m.	Half mean spring range	13ft. 9in.
Time from H.W. 6h. 8min.		Height above mean level sea	16ft. 3in.

Half mean spring range... 13ft. 9in.
Correction Table B entered with 6h. 8min. and 16ft. .... 16ft. 0in.
Correction to be *added* to sounding taken .... 2ft. 3in.

In this last problem the correction is to be added to the sounding taken because the level of the sea at the time is below its mean level at low water ordinary spring tides.

1895. On July 2, being off Crookhaven by reckoning, took a cast of the lead at 10 A.M. What correction must be applied before comparing it with chart?

We find at page 101 of the Admiralty Tide Tables that the standard port of reference for Crookhaven is Queenstown.

H.W. Queenstown (page 57)	0h. 47m. P.M.	Height of tide Queenstown ...	9ft. 8in.
Constant Crookhaven (page 101)	− 0h. 52m.	Constant Crookhaven...........	−1ft. 6in.
H.W. Crookhaven...	11h. 55m. A.M.	Height of tide Crookhaven ...	8ft. 2in.
Time of sounding ...	10h. 0m. A.M.	Half mean spring range	4ft. 10in.
Time from H.W. ...	1h. 55m.	Height above mean level sea	3ft. 4in.
Half mean spring rise Crookhaven			4ft. 10in.
Correction Table B, entered with 3ft. 4in. and 1h. 55m. ...			+1ft. 8in.
Correction to be subtracted...			6ft. 6in.

In this problem we find the half mean spring range for Crookhaven by looking out, at page 174 in the Admiralty Tide Tables, the spring rise, 9¾ft., and dividing it by two.

Tidal Streams Round the British Isles.

My remarks on this subject are based upon the Admiralty Tide Tables, and, so far as the English Channel goes, upon the experience of a friend, to whom my best thanks are due.

As we have seen, on page 256, the tidal wave from the Atlantic strikes the south-west of Ireland, and divides into three portions, one running up the English Channel, another along the west coast of Ireland and Scotland, round by the Shetlands and Orkneys, and down the east coast of England as far as the Thames, and the third enters the Irish Sea by way of St. George's Channel. The tidal streams following these waves cause the general direction of the flood to be to the eastward in the Channel, to the northward along the west coast of Ireland and Scotland, to the eastward off the Shetlands and Orkneys, and to the southward in the North Sea along the east coast of England. In the Irish Sea the flood tide enters both by the south, through St. George's Channel, and by the north, through the North Channel, between Rathlin Island and the Mull of Cantyre. It flows towards the Irish Sea while the water is rising at Liverpool, and ebbs away from it while it is falling at that port.

In order to explain more easily the direction of the tidal streams in the English Channel it is best to divide it into four parts, viz.:

(1) Westward of a line joining Land's End and Ushant.

(2) Eastward of the above, and westward of a line joining the Start and the Casquets.

(3) Between the eastern limit of the second portion and a line from Beachy Head to Point D'Ailly. The streams in this portion are commonly known as the "Channel Tides."

(4) Eastward of the line from Beachy Head to Point d'Ailly, as far as a line joining the North Foreland and Dunkerque, or even as far as a line joining the North Foreland and Texel. The tides within this area are termed the "Straits of Dover" tides.

In the first of these divisions the tides run in all directions of the compass during the twelve hours—*i.e.*, from one high water to another.

In the second division the tidal streams are also very irregular, being influenced by the regular Channel tides, by the currents flowing into and out of the Gulf of St. Malo, and by the irregular currents to the westward of Land's End. In the Gulf of St. Malo, while the water is falling at Dover, the stream sets directly into the bay, curving round Cape La Hogue, and setting south-westerly to south-easterly on both sides of the bay. The reverse takes place when the water is rising at Dover.

In the third division, or the "Channel proper," the tides are fairly regular, flowing to the eastward while the water is rising at Dover, and ebbing to the westward when it is falling at that port. The tide is slack over all this section at the same time.

The last division demands our most careful consideration. In the Straits of Dover, the Channel streams, and the stream coming from the North Sea, southward of a line joining the Wash and Texel (sometimes called the East Channel tide), meet; but their line of meeting is not stationary; it is continually shifting between Beachy Head and the North Foreland. At high water at Dover, the water is slack at Beachy Head and Point d'Ailly; but eastward of this line the stream is running easterly as far as the North Foreland, where it is also slack; while to the westward of the line there is no stream. One hour after H.W. at Dover the stream all along the Channel as far as the Royal Sovereign Light is going to the westward; but eastward of this line it still sets easterly. Two hours after H.W. at Dover, the line of separation of the streams has shifted to a line between Hastings and Tréport, *i.e.*, the water runs to the westward west of this line. Three hours after H.W. at Dover, the separation takes place at Hastings or Rye, and on the French coast at Cayeaux. Four hours after it is at Folkestone and Cape Grisnez, five hours after at the South Foreland and Calais, six hours after at the North Foreland and Dunkerque.

At low water at Dover the stream is again nearly slack at Beachy Head and at the North Foreland. One hour after it is running to the eastward, two hours afterwards it commences to run to the eastward off Hastings, three hours after at Rye and Cayeaux, four hours after at Dungeness and C. Grisnez, four and a half hours after at Dover and Gravelines, five hours after at the South Foreland and Dunkerque, and six hours after at the North Foreland and a little to the westward of Dunkerque.

It will be seen, therefore, that the direction of the stream does not generally coincide with the time of H.W. For instance, at Dover the tide flows to the eastward one and a half hours before H.W., and continues flowing until four and a half hours after H.W. at the shore.

By taking advantage of the shifting of the line of meeting, as shown above, a vessel may, providing she be at the Owers at slack water—low water at Dover—carry with her a twelve hours tide, if she can maintain a speed of eight knots through the water.

Along the coast and in the bights the streams turn one to two hours earlier than in the offing. This is useful knowledge for vessels turning down Channel. It must also be remembered that there is an indraft on both the flood and the ebb into the bights on the English coast, such as Lyme Bay, Weymouth Bay, Poole Bay, and to the eastward of the Owers.

Strong winds from the south-west and west prolong the flood stream in the Channel, and often cause the tide to rise higher. The reverse of this takes place with easterly winds.

Inside the Isle of Wight the streams may be divided into two portions, namely, one to the eastward and the other to the westward of Cowes. In the western portion, the westerly stream makes about 1 hour 20min. before until 4h. 20min. after H.W. at Dover; then the easterly stream runs until 1 hour 20min. before H.W. at Dover. In the eastern portion, the westerly stream commences about 2 hours before H.W. at Dover, and runs for five hours; the easterly stream makes about 3 hours after H.W. at Dover, and runs for 7 hours.

At Spithead, the westerly stream commences about 2 hours before H.W. at Dover, 2½ hours before H.W. at Portsmouth, and 1½ hours before H.W. at Southampton, running N.W. by N. for 5 hours, and S.E. by S. 7 hours. This is the principal cause of the double high tides at those ports and elsewhere inside the Isle of Wight, which double tides are felt at Christchurch, Poole, and Swanage, and their effects even as far as Weymouth.

NORTH SEA.

In the North Sea, when it is H.W. at Dover, the flood tide is still making at the mouth of the Thames. It is slack water nearly to 54° N. A northerly stream makes from 54° to 56° N., and a southerly from 57° to 59° N.

One hour after H.W. at Dover it is slack water at the mouth of the Thames. The current sets to the northward at the rate of from 1 to 2 knots to 54° N., where it assumes a more westerly direction. On the parallels of 55° and 56° it is more or less slack water, but there is a southerly set to the northward of this.

Two hours after H.W. at Dover, ebb tide at the mouth of the Thames, otherwise the same as at one hour.

Three hours after H.W. at Dover, ebb tide at the mouth of the Thames. A northerly current to 53° N., turning to the westward between 53° and 54° N., and westerly and southerly currents from 54° N. to 57° N.

Four hours after H.W. at Dover, ebb tide at the mouth of the Thames. The northerly current continues to 54° N.; 55° N. westerly currents from 8° to 3° E., and southerly currents from 2° E. to the coast of England. At this time we have a southerly current along the coast of Scotland and England from St. Abb's Head nearly to the Wash, and a northerly current along the coast of Scotland from the Firth of Forth to the Orkneys.

Six hours after H.W. at Dover, ebb tide at the mouth of the Thames, slack water off Harwich. Northerly stream, except close into the coast of France and Belgium, where it runs to the south-westward. The southerly stream along the east coast of England now comes down as far as the Wash.

Five hours before H.W. at Dover, slack water at the mouth of the Thames. Along the coast of England we have a southerly stream from the river Tyne to the Straits of Dover, and to the southward of 53° N. From the parallel of 54° N. to 56° N. the stream is easterly, and further to the northward northerly.

Four hours before H.W. at Dover, flood tide at the mouth of the Thames. To the southward of 53° N. the stream is southerly, and in the remainder of the North Sea it runs from easterly on 54° N. to northerly on 56° N. and to the northward. There is a northerly stream from the river Tyne along the coast to Fraserburgh.

Three hours before H.W. at Dover, flood tide at the mouth of the Thames, and similar tidal streams to those which existed one hour before. A north-easterly stream close to French coast off Dunkerque.

Two hours before H.W. at Dover, flood tide at the mouth of the Thames. Very much the same state of affairs exists as one hour previously.

One hour before H.W. at Dover, flood tide at the mouth of the Thames, and nearly the same as at the previous hour.

THE IRISH SEA.

One hour after low water in Morecambe Bay the tidal streams begin to flow into the Irish Sea to the southward through the north channel, and to the northward from a line joining Carnsore Point and the Bishops. The current from the south runs up until it passes Holyhead, when the greater portion sweeps off to the eastward, to the southward of the Isle of Man, and with the current from the northward, it flows into Morecambe Bay. The smaller western portion of the current from the south runs along the coast of Ireland till it is met and neutralised by a portion of the current from the north to the westward of the Isle of Man, where there is a large surface of water, some twenty or thirty miles in diameter, where no current is perceptible, though the rise and fall of the water is considerable, amounting at Springs to 16 feet.

At H.W. in Morecambe Bay there is slack water in the Irish Sea, and then the currents are reversed and run in opposite directions to those in which they flowed while the water was rising.

The Bristol Channel.

When the water is falling in Morecambe Bay there is a north-easterly current off the N.W. coasts of Cornwall and Devon, and an easterly current off the south coast of Wales. When it is H.W. or L.W. at Morecambe Bay we have slack water in this channel, and when the tide is rising in that bay the above-mentioned currents are reversed in direction.

The West Coast of Scotland.

The tides run with great velocity; but as the bays, lochs, &c., are very deep, and the rise and fall of the tide slight, it is more important to know the time of the change of direction and the rate of the streams than the times of high and low water. The stream runs about six hours in each direction, flowing to the northward from half an hour to an hour after low water at Liverpool, and to the southward from half an hour to an hour after high water at that place.

Velocity of Tidal Streams.

It is quite impossible to give the velocity of the tidal streams. Roughly, on our side of the English Channel, the rate varies from 1·5 to 2·5 knots. On the French side, particularly near the Gulf of St. Malo, it is much more rapid.

In the Irish Sea, also, the velocity of these streams is from 1·5 to 2·5 knots. In the north channel, however, it often attains a rate of 5 knots.

The navigator, when he cannot ascertain the place of his ship by other means in home waters, should allow for the tidal streams, as above indicated, in working out his dead reckoning; but he should constantly check the positions thus ascertained by the use of the lead.

The Admiralty Tide Tables should be on board every vessel coasting in our narrow seas. The information given in them is compiled from the best sources, and is as reliable as it is possible for it to be under the ever-varying conditions of wind and weather which exist in the British Isles.

EXTRACTS FROM ADMIRALTY TIDE TABLES, 1895.

Month.	Day.	Place.	HIGH WATER.				Half mean spring range.
			A.M.		P.M.		
			Time. h. m.	Height. ft. in.	Time. h. m.	Height. ft. in.	ft. in.
March	1	Devonport	8 9	14 6	8 24	14 2	7 9
,,	1	Portsmouth	2 0	13 0	2 15	12 10	6 9
April	9	Greenock	11 39	10 1	—	—	4 10
August	3	Devonport	3 52	12 8	4 23	13 10	7 9
Nov.	1	Brest	2 39	17 8	2 54	18 0	9 6
March	18	Brest	8 35	13 0	9 10	12 4	9 6
,,	12	Liverpool	—	—	0 2	30 0	13 9
July	2	Queenstown	0 13	9 8	0 47	9 8	5 10

Port.	Constants.		Standard port for reference.
	Time. h. m.	Height. ft. in.	
Penzance	− 1 13	+ 1 0	Devonport.
Portland Breakwater	− 4 40	− 6 0	Portsmouth.
Oban	+ 5 20	+ 2 0	Greenock.
Crookhaven	− 0 52	− 1 6	Queenstown.

CHAPTER XXV.

THE LAW OF STORMS.

THE Law of Storms is a large and interesting subject, leading to the study of that little known science—Meteorology. It is very necessary that every seaman should, for the safe conduct of his ship, be thoroughly conversant with the nature of circular storms.

The earth is surrounded by an envelope of air which is from 45 to 200 miles in height according to different authorities. It consists of minute particles, each of which is independently attracted to the earth by gravitation—that is to say, they have weight. Air is an elastic body, capable of expansion and contraction. The stratum lying nearest to the earth is compressed by the weight of the air above it, and is consequently denser and heavier than any other portion of the atmosphere. That the air has weight was first proved by an Italian scientist named Torricelli (a pupil of Galileo) in A.D. 1643. He took a tube about 36 inches long, hermetically sealed at one end, and filled it completely with mercury. Placing his finger over the open end, he inverted the tube and plunged it into a bowl of mercury. His finger being removed, the mercury in the tube sank to a height of about 30 inches above the level of the mercury in the bowl. It did not run out of the tube altogether, because there was a vacuum above the mercury in the tube, and therefore no pressure of air upon its surface, while upon the surface of the mercury in the bowl was the pressure of the superincumbent mass of the earth's atmosphere. Now, a column of mercury 30 inches long, one inch in diameter—that is, one square inch in sectional area—contains 30 cubic inches of mercury weighing 15lb., and this is evidently the weight of a column of the atmosphere having the same sectional area at the level of the sea, because the two columns balance each other. If water, which is 13½ times lighter than mercury, be used instead, the tube must be 30 × 13½ inches in length, or 33¾ feet.

The ordinary mercurial barometer is constructed on the principle just described. It measures the weight of the atmosphere, falls when the weight of the atmosphere diminishes by expansion, and rises when it becomes denser by contraction.

The aneroid, or dry barometer, consists of a corrugated metal box, from which most of the air has been pumped. This box is bolted to the framework of the instrument, and is kept from collapsing by an internal spring, which expands when the atmospheric pressure decreases, and is forced inwards when the atmosphere becomes heavier. A chain or lever is attached to the top of the box, and so connected with the hand on the dial as to cause it to register the variation of atmospheric pressure. This instrument is in many respects superior for marine purposes to the mercurial barometer. It is lighter and more portable, it indicates changes of pressure more quickly, and is more easily read. But it does not record absolute pressure, nor does it preserve its accuracy, as the parts may rust or the spring get weaker; therefore its readings must occasionally be checked by the mercurial barometer. However, to the seaman the variation in pressure of the atmosphere is of much more consequence than its absolute weight, and therefore the aneroid is more useful to him.

The scale of a barometer is divided into inches, tenths, and hundredths of an inch. In first class instruments the readings are taken by means of a vernier.

The Thermometer is an instrument which measures heat, and is in no way affected by the changes of pressure in the atmosphere. It consists of a glass tube of very small bore, with a bulb at the end, which is filled with mercury and heated to expel all the air. When quite full of warm mercury the open end is hermetically sealed. As the mercury cools it contracts, and leaves an empty space at the top of the tube. The thermometer is then placed in a mixture of ice and water, for melting ice has always the same temperature. The point to which the mercury falls is marked, and is called the freezing point. Then it is put into boiling water, and immediately the mercury expands and rises in the tube to a certain point which is marked; this is the boiling point.

In English thermometers the boiling point is 212°, and that of freezing 32°; the space between these two is then divided into 180 equal parts, which are degrees of temperature. Below freezing point the scale is continued, but except in specially made thermometers extreme readings are very unreliable.

England, France, and Germany use thermometers cut to different scales.

England: Fahrenheit; freezing point, 32°; boiling point, 212°; and difference, 180°.

France: Centigrade; freezing point, 0°; boiling point, 100°; and difference, 100°.

Germany: Réaumur; freezing point, 0°; boiling point, 80°; and difference, 80°.

Wind is air in motion, and is generally caused by unequal expansion of the atmosphere, due to difference of temperature. The wind is named according to the point of the compass from which it blows, and varies in strength. Beaufort's scale of wind pressure, which is generally adopted in England, is as follows:

BEAUFORT'S SCALE.

Hourly Velocity.	Scale Number.	Character of Wind.
	0	Calm.
1 mile	1	Light airs.
2 to 3 miles	2	Light breeze.
4 to 7 ,,	3	Gentle breeze.
9 to 15 ,,	4	Moderate breeze.
15 to 18 ,,	5	Fresh breeze.
19 to 22 ,,	6	Strong breeze.
23 to 28 ,,	7	Moderate gale.
28 to 40 ,,	8	Fresh gale.
40 to 48 ,,	9	Strong gale.
48 to 56 ,,	10	Whole gale.
57 to 80 ,,	11	Storm.
80 to 100 ,,	12	Hurricane.

In the first chapter I explained to what causes the trade winds are due. There are other winds which are fairly constant, such as the prevailing westerly winds on the polar sides of the 40th parallels of latitude in both hemispheres. But the subject with which I am here dealing is quite a different matter. A gale of wind may be a prevailing wind of extraordinary force; but not infrequently it is part of a circular storm, and it is upon the characteristics of this latter phenomenon that I would dwell.

In the neighbourhood of the equator, owing to some sudden expansion of the atmosphere or other cause, the air over a small area becomes much lighter than that surrounding it, which rushes in to restore the equilibrium from all sides, and as the result a rotatory motion is commenced. At first this occupies only a small space, but, like a snowball, it increases and developes as it travels, till its diameter may become as much as 1000 miles.

In the North Atlantic Ocean, cyclones, or circular storms, commence in from 6° to 8° N. latitude. They travel to the westward and north-westward till they arrive in the neighbourhood of the West Indies, where they curve to the northward and then to the north-eastward, moving across the Atlantic almost along the path of the Gulf Stream, increasing in diameter and gradually diminishing in energy till they at last blow themselves out in the vicinity of the British Isles.

In the southern hemisphere a cyclonic storm originates near the equator, travels to the westward and south-westward, and recurves to the southward and south-eastward.

One peculiarity of these storms, worthy of particular attention is that in the southern hemisphere their rotatory motion is in the same direction as that in which the hands of a watch move, whilst in the northern hemisphere they revolve in the opposite direction. Another fact to be noted is that the wind does not blow in a circle round the centre of the storm, but curves in spirally, so that when facing the gale in the northern hemisphere, the centre lies at an angle of about 10 points to the right of the direction of the wind. In the southern hemisphere it will lie to the left at a similar angle.

When the mariner encounters a cyclone, he must use every endeavour to avoid getting into the centre of it, where an unnatural calm is alternated with the most furious gusts from all points of the compass, an awful sea is running, and in which ships are usually dismasted and often founder. He has two means whereby he may form an accurate judgment as to his position with regard to this centre, and as to its direction and rate of progression. The first is the barometer, which slowly and steadily falls as the storm centre approaches; the second is the changes in the direction of the wind, which indicates the position of the ship with regard to the path of the storm.

Let Fig. 39 represent a cyclone in the northern hemisphere which is travelling to the north-eastward. In speaking of a circular storm the right hand semi-circle is that half of the storm lying to the right of the direction in which it is travelling. The left-hand semi-circle, of course, is the other. In the northern hemisphere the right-hand semi-circle is sometimes called the dangerous semi-circle, and that to the left the navigable semi-circle. In the southern hemisphere the left-hand semi-circle is the dangerous one. The ship, at A, will have a falling barometer and rising gale from the eastward; it is to be presumed, then, that the centre bears from her S.S.W. The glass still falls and the wind shifts to E.N.E. as at B.—that is to the left. This shows that she is in the left-hand semicircle, and must be hove to on the port tack, because she

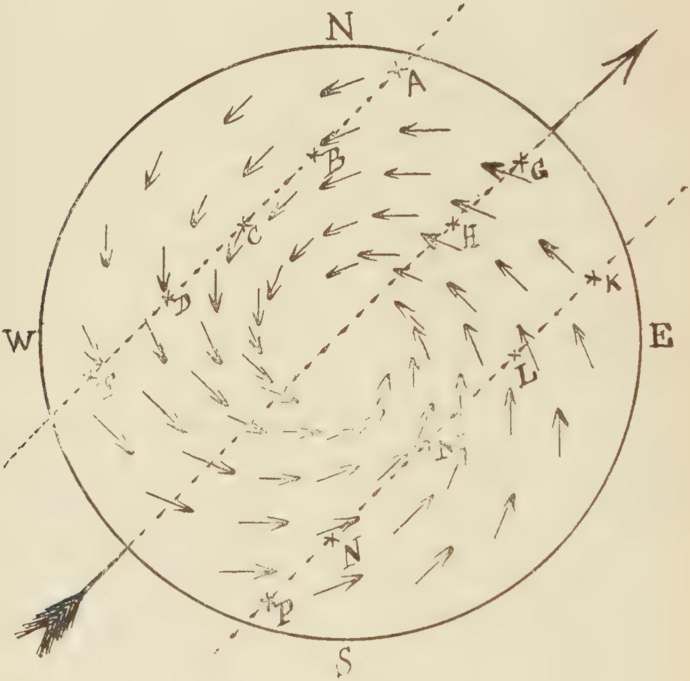


Fig. 39.

will not then run the risk of being taken aback. As he storm progresses the ship will assume the position C, having the wind at N.E.; then D, with a rising barometer, the wind falling, and weather improving; till when the storm passing her leaves her at F, with the wind N.N.W. and fine weather, she may make sail and bear away on her course.

Suppose, however, she gets the wind first at E.S.E., as in the position G. The glass falls rapidly without changing its direction and the wind increases in violence with terrific squalls. It is clear that she is right in the path of the storm, and to avoid passing through the centre she must run, keeping the wind aft till the barometer begins to rise and the wind to shift, when she may be hove-to on the port tack.

If a vessel is in the right-hand semi-circle, as at K, the advance of the cyclone will cause the wind to shift to the right. Thus at K it is S.E.; at L, S.S.E.; at M, S.S.W.; at N, W.S.W.; and at P, W. In this case she must be hove-to on the starboard tack.

The rule for heaving-to which holds good for all circular storms in any part of the world is: In the left-hand semi-circle the wind shifts to the left, heave-to on the port tack. In the right-hand semi-circle the wind shifts to the right, heave-to on the starboard tack.

It may happen, however, that a ship is travelling faster than the cyclone and in the same direction. Then the wind shifts to the left in the right-hand semi-circle, and *vice versâ* in the left-hand semi-circle. In such a case the barometer will indicate whether you are approaching or receding from the centre.

In the neighbourhood of the West Indies, where cyclones alter their direction or recurve, it is, of all things, necessary to watch the barometer most closely, as there the changes in the direction of the wind are of little use to indicate the track of the storm, whilst the glass will infallibly fall if you are nearing the centre, or rise if receding from it.

Cyclones in the North Atlantic most frequently occur from July till the beginning of November.

The typhoons in the China seas are usually experienced between July and November. Off the coast of Japan, however, they may only be expected between August and October. Their usual track is W.N.W. till they approach the land, when they recurve to the northward and north-eastward till they sometimes reach the Aleutian Islands. Some, however, recurve to the southward, and may occur at any season. September is the worst month.

In the Bay of Bengal the storms commence in about 10° N. latitude and travel to the W. and N.W. during the months of May and June and during October, November, and December, those occurring in the two latter months being the more severe. Their rate of progress is from three to fifteen miles per hour.

In the South Indian Ocean cyclones occur between December and April. They commence in about 10° S. latitude and 95° E. longitude, and travel W.S.W. till they arrive to the southward of Mauritius or Reunion, where they recurve to the S.W., in latitude 25° S. turn south, then they further recurve to the S.E. and E.S.E., and generally exhaust themselves in about 33° S. latitude.

In the South Pacific they commence to the eastward of the Low Archipelago, that is, in about 135° W. longitude, and from the Equator to about 25° S. latitude. They follow a westerly course till they reach the Barrier Reefs of Australia, then recurve to the southward. From December to March these storms may be encountered.

In the Arabian Sea there are two tracks made by cyclones. The first begins about the Laccadives, and follows a W.N.W. course till exhausted on the Persian coast. The second commences between the Laccadives, Maldives, and the Malabar coast, and follows the line of the shore to the N.N.W. These latter sometimes are very severe. During April and May and October and November such storms may be expected.

The monsoons in the Indian Ocean are more of the character of trade winds. The large expanse of land situated to the northward of the Indian Peninsula when heated by an almost vertical sun for many months causes the atmosphere in that locality to expand and rise; as a consequence the air from the Indian Ocean is drawn in to fill the vacuum. Thus we have the S.W. monsoon.

CHAPTER XXVI.

GENERAL REMARKS.

BEFORE sailing, the navigator should be provided with all the proper instruments, charts, and books.

The necessary instruments are Compasses, azimuth and ordinary; Chronometers; Leads and Logs, patent and ordinary; and a reliable Sextant.

The charts should be of recent date, and must embrace all that part of the ocean intended to be sailed over, or to which the ship might be driven by stress of weather. The most recent plans of harbours and roadsteads are indispensable. Admiralty charts are perhaps the most reliable; but the blue-backed charts are admirably drawn, and very often even superior in distinctness to the official ones. In connection with charts, dividers, parallel rulers, protractors, pencils, and india-rubber must not be forgotten. A pair of single-handed or calliper dividers and a station pointer would be found very useful. One pair at least of Field's parallel rulers should be taken; they are used for laying off true courses directly from the meridian instead of having to apply the variation, which we are compelled to do when using a magnetic chart. Care should be taken to examine the parallel rulers to see that they are not warped, and that they work easily.

With regard to compasses there are three things to be considered. The first is the instrument itself; the second, its position on board ship; and the third, its compensation if necessary.

A good compass should possess in a high degree sensibility and steadiness, and be of simple construction. By sensibility is meant that the needle should have great directive force, or, in other words, should submit itself freely to the magnetic force of the earth; at the same time it should not be too lively, because in a seaway, when the ship has much motion, the card would swing so that it might become difficult to steer a course with any degree of accuracy, but steadiness must not be obtained by friction or weighting the card at the cost of directiveness and sensibility.

The best compass needles are thin compound bars of steel, two or more of which should be fastened by their edges to the under-surface of the compass card, as it has been found that one magnet does not possess nearly the directive force of two or more which together equal it in weight, and another thing is that if only one needle is used it must be placed on its flat, which sometimes causes an instrumental error if the line of polarity of the magnet does not run parallel to its edges.

The magnetic direction of the system of needles should coincide with the north and south points of the compass card exactly. The points of suspension of the gimbals of the bowl should be on the same plane when the ship is upright as the point of suspension of the card. The compass card should be in the centre of the bowl, and accurately marked. Brass slides ought to be fitted to the frame that carries the needles to adjust for dip. The navigator must see that spare cards, caps, and pivots are supplied for each compass. The cap should have a hard stone to hang on the pivot, which is of hardened steel, pointed, but not too sharply. The lubber's line should be distinct, vertical, and exactly amidships.

The standard compass on board any ship must be placed exactly on the midship line, and as far away from vertical iron as possible. It should also command a view of the horizon in every direction. It is fitted with an eye piece and sight vane for the purpose of taking bearings. Sometimes in the centre of the glass cover there is a hole, into which an upright brass rod, called a stile, may be fixed for the purpose of taking the sun's bearing by the stile's shadow cast on the compass card.

Among the books to be provided are Tables of Logarithms, Nautical Almanacs, Admiralty Tide Tables, Sailing Directions, Burdwood's and Blackburne's Azimuth Tables, Johnson's Tables and Bearings of Principal Bright Stars, and the book of the International Code of Signals.

A good sextant is an absolute necessity, and it is advisable to purchase one from a reliable maker. It may cost a little more, but the money is well spent. No amateur should trust to his own unguided opinion in so important a matter. I would say the same regarding the choice of chronometers.

When proceeding to sea, have little faith in the deviation card with which the compass adjuster has provided you. Bear in mind that an iron ship acquires transient induced magnetism in any position in which she may have been lying, and parts with it more or less quickly. I have it on trustworthy authority that in a certain P. and O. steamer, homeward bound, after leaving the Mediterranean, where she had steered westerly courses, the standard compass was deflected 10° from this cause when she headed north; but in the course of two or three days the needle had nearly resumed its normal position. It is absolutely necessary, therefore, to find your compass error by azimuths and amplitudes morning and evening. Should you have to alter course considerably during the night, it is a needful precaution on the preceding afternoon to swing your ship's head round to the course to be steered and ascertain the compass error by an observation. As a matter of fact, the compasses require constant care and watching, which is well repaid when no observation can be taken.

The chronometers should be placed as nearly as possible in the centre of motion of the ship and securely packed with cotton wool. They should be wound up at a regular hour each day; eight o'clock in the morning is generally found the most convenient time. Eight-day chronometers may be very useful, but there is great risk of letting them run down if they are only wound once a week; whereas it becomes a matter of habit to the navigator, who sees to his chronometers before breakfast every day.

When coasting, every opportunity should be seized of fixing the ship's position on the chart by cross-bearings, doubling the bow angle, &c., particularly when it is likely to come on thick.

In the open sea it is customary to take morning and afternoon sights, and to ascertain the compass error on each course that may be steered. When approaching the land or any other danger the three L's must not be neglected, namely, Look-out, Log, and Lead.

With regard to the Rule of the Road,* if it is your duty to give way, do so in plenty of time; you have no right to cause anxiety to the officer in charge of an approaching ship by hanging on too long and making close shaves. Many ships have been lost through nervousness caused by laxity in this respect; making a man lose his head and doing the wrong thing at a critical moment.

Steam ships are bound to get out of the way of vessels under sail, and officers in charge of them may use their own judgment in doing so; therefore, a sailing vessel should not alter her course when being approached by a steamer. A steam vessel in which I was serving was nearly run down by a barque off the coast of Spain because she ported her helm instead of allowing us to get out of her way, which we could easily have done by crossing her bows. As it was, the proverbial biscuit could easily have been thrown on board her as she rushed under our stern.

In narrow channels, however, it is the duty of sailing vessels, on being signalled to by a large steamer, to keep out of her way.

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* See Regulations issued by Board of Trade, which are contained in the signal book.

STEAM YACHT "DEVIATION."

How Positions are Determined on Plate IV.

POSITION.

1, A.—South Foreland N.N.W. (magnetic), distant 3¼ knots.

2, B.—Dungeness Lower Lighthouse N. 39° W. (magnetic), 3½ knots.

3, C.—Royal Sovereign Light Vessel N. 6° W. (magnetic).

4, D.—Dunnose Point N. 13° W. (magnetic), distant 8¾ knots.

5, E.—{Portland High Light N. 38° W. (magnetic).
Anvil Point Light N. 24° E. (magnetic).

6, F.—Start Light N. 21° W. (magnetic), distant 8 knots.

7, G.—By chronometer observation.

8, H.—Lizard Head N. 11° W. (magnetic), distant 7½ knots.

9, I.—By meridian altitude of sun and bearing of Lizard Point N. 71° E. (magnetic).

10, J.—Wolf Light House N. 24° E. (magnetic), distant 2 knots.

11, K.—{Longships Light House bore N. 76° E.
Wolf Light House bore S. 34° E.

12, L.—By solution of double chronometer problem.

13, M.—By dead reckoning from position L., and allowing a set of 1½ knots per hour to the eastward since 9.30 p.m.

At 6.15 the speed is reduced because the depth of water is only 29 fms., and an examination of the chart in this locality shows that we must be getting close in.

14, N.—{St. Ann's Head bore N. 56° E. (magnetic).
Linney Head bore S. 81° E. (magnetic).

31
Aug. 16
Aug 15
Sept 29
Days 44

CRUISE OF STEAM YACHT "DEVIATION."—Plates A, B, C.

Elements from Nautical Almanac, 1895.

SUN.

Date.	Declination Apparent Noon.	Variation in one hour.	Declination Mean Noon.	Semi-diameter.	Equation of Time to be added to Apparent Time.		Sidereal Time.
					m. s.	Var. in one hour.	h. m. s.
Aug. 15	14° 3′ 55″·44 N.	46″·86	14° 3′ 58″·8 N.	15′ 50″	4 20·36	·481s.	9 34 34·21
.. 16	13° 45′ 3″·9 N.	47″ 42	13° 45′ 7″·1 N.	15′ 50″·1	4 8·56	·502s.	9 38 30·77

Polaris.—See Table 21 Inman; Table XVII. Norie.

Antares.—Right Ascension, 16h. 23m. 1s. Declination, 26° 12′ 12″ S.

Elements from Admiralty Tide Tables, 1895.

Pembroke on August 17th.—H.W., 2h. 23m. A.M.; Height, 17ft. 1in.; Half Mean Spring Range, 11ft. 3in.

Milford. Constants — 0h. 16m. + 1ft. 3in.

3-56.56
180
236.56
44
9 4 6 2 4
9 4 6 2 4
10 4 0 8,6 4
M S
173-28.64
9- 38-30.77
9- 211-59.41
180
3- 31

-6-
-3 i 0
10 4 c

CRUISE OF CUTTER YACHT "MAGNETIC."—PLATES D TO H.

ELEMENTS FROM NAUTICAL ALMANAC, 1895.

SUN.

Date.	Declination Apparent Noon.	Variation in one hour.	Declination Mean Noon.	Semi-diameter.	Equation of Time to be subtracted from Apparent Time.		Sidereal Time.
					m. s.	Var. in one hour.	h. m. s.
May 26	21° 8′ 9″·3 N.	26″·02	21° 8′ 10″·7 N.	15′ 49″·1	3 13·07	·267s.	4 15 13·03
.. 27	21° 18′ 22″·9 N.	25″·11	21° 18′ 24″·2 N.	15′ 48″·9	3 6·43	·286s.	4 19 9·59
.. 28	21° 28′ 14″·4 N.	24″·18	21° 28′ 15″·6 N.	15′ 48″·8	2 59·33	·305s.	4 23 6·15
.. 29	21° 37′ 43″·5 N.	23″·24	21° 37′ 44″·6 N.	15′ 48″·6	2 51·79	·323s.	4 27 2·71

MOON.

Date.	Right Ascension.	Declination.	Semi-diameter.		Horizontal Parallax.	
			Noon.	Midnight.	Noon.	Midnight.
May 29	At 5h. .. 9h. 17m. 39·89s. Var. in 10min., 23·257s. increasing.	At 5h. ... 18° 34′ 36·6″ N. Var. in 10min., 132″·83, decreasing.	16′ 13″·2	16′ 12″·9	59′ 25″·6	59′ 24″·6

STARS.

Date.	Names.	Right Ascension.			Declination.		
		h.	m.	s.			
May 31	Arcturus	14	10	54·86	19°	43′	33″·4 N.
,,	Vega	18	33	25·4	38°	40′	54″·9 N.
	Altair	19	45	41·89	8°	35′	19″·3 N.
	Markab	22	59	32·98	14°	38′	23″·9 N.

ELEMENTS FROM ADMIRALTY TIDE TABLES, 1895.

Date.	Place.	H.W. A.M.		H.W. P.M.		Height A.M.		Height P.M.	
		h.	m.	h.	m.	ft.	in.	ft.	in.
May 26	Dover	—	—	0	9	—	—	18	0
,, 27	,,	0	34	0	59	18	0	17	11
,, 28	,,	1	24	1	50	17	9	17	7
,, 29	,,	2	16	2	43	17	4	17	0
,, 30	,,	3	11	3	40	16	9	16	4

EXAMPLES.

In the following examples, as far as possible, every form in which the problems can present themselves is given. After each question, when required, are inserted the necessary data from the Nautical Almanac. Readers are, however, advised to obtain a Nautical Almanac for 1895 and look out the necessary information for themselves.

For the general examples—viz., the logs of the yachts "Deviation" and "Magnetic," the requisite extracts from the Nautical Almanac are given on pages 294, 295, and 296.

Logarithms.

Find the logarithm of ... 8·26 = 0·916980
" " 101·1 = 2·004751
" " ·0101 = $\bar{2}$·004321
" " 10·964 = 1·039970
" " 1279·8 = 3·107142
" " ·86 = $\bar{1}$·934498
" " 5017924 = 6·700524
" log sine of 27° 28′ 45″ = 9·664102
" " cosine of 74° 11′ 35″ = 9·435202
" " cosecant of 94° 27′ 8″ = 10·001313
" " tangent 45° 1′ 10″ = 10·000295
" " ½ haversine 138° 12′ 40″ = 4·970458
" " haversine or horary angle } 10h. 28m. 40s. = 9·982641

Of what number is	3·769844	the logarithm	5886·33 nearly.
,,	1·000064	,,	10·0015 nearly.
,,	$\bar{2}$·968432	,,	·092989 nearly.
,,	0·769543	,,	5·882 nearly.
,,	$\bar{1}$·010000	,,	·102329 nearly.
,,	7·900000	,,	79,432,735 nearly.
,,	$\bar{5}$·000100	,,	·00001 nearly.
angle is	9·898642	the log sine	52° 21′ 30″
,,	10·473510	,, ,, tangent	71° 25′ 18″
,,	10·321768	,, ,, cotangent	25° 29′ 11″
,,	7·231486	,, ,, cosine	89° 54′ 9″
,,	8·790768	,, ,, haversine or horary angle	28° 46′ 53″
hour angle is (East of Mer.)	9·428751	,, ,, ,,	19h. 50m. 23s.
,, (West of Mer.)	7·794361	,, ,, ,,	0h. 36m. 13s.

Multiply	1789	by	2·635	by logs.	=	4714·01 nearly.
,,	·04672	,,	21798	,,	=	1018·403 ,,
,,	·02	,,	·0006	,,	=	·000012
,,	7984202	,,	·00562	,,	=	44871·1 nearly.
,,	11·0001	,,	·001	,,	=	·0110001
,,	90009	,,	·0909	,,	=	8181·82 nearly.
Divide	7896	,,	·01482	,,	=	532793·7 ,,
,,	·001	,,	78	,,	=	·00001282 ,,
,,	178596	,,	371·462	,,	=	480·793 ,,
,,	1·79648	,,	·000486	,,	=	3753·6 ,,
,,	1000·12	,,	985·432	,,	=	1·0149 ,,
,,	7·964321	,,	70	,,	=	·113776 ,,

Parallel Sailing.

1. A ship sails due West 148 miles on the parallel of 51° 28′ N. latitude. Required the difference of longitude she has made?

Difference of longitude, 3° 57′·6 W.

2. A ship in latitude 41° 12′ S. and longitude 161° 25′ E. sails East (true) 276 miles. Required her position?

Latitude 41° 12′ S., longitude 167° 31′·8 E.

3. A ship in latitude 55° 10′ N. changes her longitude 4° 56′ to the eastward. What number of miles did she sail?

169·1 miles.

4. Find the departure due to a difference of longitude of 17° 28′ on the parallel of 35° 14′ S.

856·1 miles.

5. A ship in changing her longitude 4° 15′, sails West (true) 144 miles. In what latitude is she?

Latitude 55° 37′.

6. A ship sails 721 miles on a certain parallel of latitude, and changes her longitude 17° 40′. What is the latitude?

Latitude 47° 8′ 30″.

7. A ship in latitude 12° 28′ N., and longitude 179° 14′ W., sails West (true) 340 miles. Required her position?

Latitude, 12° 28′ N.; longitude, 174° 57′·8 E.

8. A ship sails East (true) 198 miles and changes her longitude by doing so from 177° 12′ E. to 177° 12′ W. What latitude is she in?

Latitude 53° 53′ 45″.

9. A ship in latitude 62° 15′ N. sails from longitude 10° 20′ E. to longitude 1° 49′ W. What course and distance has she made?

West 339·3 miles.

Mercator's Sailing.

1. Find the course and distance from A to B.

Latitude A...... 27° 12′ N. Latitude B...... 51° 13′ N.
Longitude A ... 15° 12′ W. Longitude B ... 61° 28′ W.
N. 55° 42′ 30″ W., 2557 miles.

2. Find the course and distance from A to B.

Latitude A...... 51° 20′ N. Latitude B...... 2° 18′ S.
Longitude A ... 18° 12′ W. Longitude B ... 29° 15′ W.
S. 10° 3′ 15″ W., 3268 miles.

3. Find the course and distance from Hawaii, Sandwich Islands, to Cape St. Lucas, California.

Hawaii	Latitude, 19° 45′ N.;	longitude, 155° 5′ W.
Cape St. Lucas ...	,, 22° 53′ N.;	,, 109° 55′ W.

N. 85° 44′ 15″ E., 2529 miles.

4. Find the course and distance from Cape Guardafui, East Coast of Africa, to Arthur Head, Perth, Australia.

Cape Guardafui ...	Latitude, 11° 50′ N.;	longitude, 51° 16′ E.
Arthur Head	,, 32° 3′ S.;	,, 115° 45′ E.

S. 54° 37′ 30″ E., 4548 miles.

5. Find the course and distance from Ushant to Pernambuco.

Ushant	Latitude, 48° 28′ N.;	longitude, 5° 4′ W.
Pernambuco.........	,, 8° 3′ S.;	,, 34° 52′ W.

S. 25° 5′ 15″ W., 3744 miles.

6. Find the course and distance from Cape Horn to the Cape of Good Hope.

Cape Horn	Latitude, 55° 59′ S.;	longitude, 67° 16′ W.
Cape of Good Hope.	,, 34° 21′ S.;	,, 18° 29′ E.

N. 69° 58′ 30″ E., 3791 miles.

FOR DEVIATION CURVE.

1.

Ship's Head by Standard Compass.	Bearing of distant object by Standard Compass.	Deviation.	Ship's Head by Standard Compass.	Bearing of distant object by Standard Compass.	Deviation.
North.	S. 79° E.	1° E.	South.	S. 76° E.	2° W.
N.E.	S. 84° E.	6° E.	S.W.	S. 72° E.	6° W.
East.	S. 88° E.	10° E.	West.	S. 68° E.	10° W.
S.E.	S. 82° E.	4° E.	N.W.	S. 73° E.	5° W.

2.

North.	N. 7° E.	2° W.	South.	N. 4° E.	1° E.
N.E.	N. 16° E.	11° W.	S.W.	N. 7° W.	12° E.
East.	N. 15° E.	20° W.	West.	N. 16° W.	21° E.
S.E.	N. 14° E.	9° W.	N.W.	N. 4° W.	9° E.

3.

North.	East.	1½° W.	South.	N. 88° E.	½° E.
N.E.	N. 80° E.	8½° E.	S.W.	S. 84° E.	7½° W.
East.	N. 72° E.	16½° E.	West.	S. 76° E.	15½° W.
S.E.	N. 82° E.	6½° E.	N.W.	S. 83° E.	8½° W.

4.

North.	S. 9° W.	1° E.	South.	S. 11° W.	1° W.
N.E.	S. 24° W.	14° W.	S.W.	S. 3° E.	13° E.
East.	S. 38° W.	28° W.	West.	S. 18° E.	28° E.
S.E.	S. 23° W.	13° W.	N.W.	S. 2° E.	12° E.

5.

North.	S. 76° W.	1½° E.	South.	S. 79° W.	1½° W.
N.E.	S. 83° W.	5½° W.	S.W.	S. 72° W.	5½° E.
East.	N. 89° W.	13½° W.	West.	S. 65° W.	12½° E.
S.E.	S. 85° W.	7½° W.	N.W.	S. 70° W.	7½° E.

6.

North.	South.	1½° W.	South.	S. 3° E.	1½° E.
N.E.	S. 14° W.	15½° W.	S.W.	S. 19° E.	17½° E.
East.	S. 30° W.	31½° W.	West.	S. 34° E.	32½° E.
S.E.	S. 15° W.	16½° W.	N.W.	S. 16° E.	14½° E.

DAY'S WORK.

No. 1.

H.	K.	T.	Course.	Lee Way.	Wind.	Force.	Deviation.	Remarks.
1	7	—	N.E.	1	N.N.W.	5	12° E.	Noon. A point of land in latitude 14° 21′ S., and longitude 47° 10′ W., bore by standard compass W. by N. distant 12 miles, the ship's head being N.E. Variation of the compass, 11° W.
2	7	—						
3	8	—	N.E. ½ N.	¾	„	„	11° E.	
4	8	—						
5	7	—	N.E. by N.	¾	N.W. by N.	„	„	
6	7	—	„					
7	7	—	W. by S.	¾	„	„	3° W.	
8	7	—						
9	6	—	W.S.W.	½	N.W.	4 to 5	5° W.	
10	6	—						
11	6	—						
12	6	—	„	„	„	„	„	
1	6	—	West.	¾	N.N.W.	4 to 5	2° E.	
2	6	—	W.N.W.	¾	North.	„	6° E.	
3	7	—	„					
4	7	—	E.N.E.	1¼	„	„	5° E.	
5	6	—						
6	6	—						
7	5	—	E. by N.	1	N. by E.	4	2° E.	
8	4	—						
9	4	—						
10	3		East.	½	N.N.E.	2	2° W.	A current set the ship during the last 24 hours, correct magnetic W.S.W. 18′.
11	3	—						
12	3		„	„	„	„	„	

Latitude 14° 21′ 18″ S. Longitude 46° 47′ 0″ W. Course and distance made good, S. 89° E., 22·4 miles.

No. 2.

H.	K.	T.	Course.	Lee Way.	Wind.	Force.	Deviation.	Remarks.
1	7	2	N.E.	Nil.	W.N.W.	5	19° W.	Noon. A point of land in latitude 50° 10′ S., longitude 47° 40′ E., bore S.E. ½ S. by compass, distant 9 miles, the ship's head being N.E.
2	6	8						
3	7	5						
4	8	5			,,	6		
5	9	—						
6	9	5	N.E. by N.	Nil.	,,	7	17° W.	
7	10	—						Variation 6° E.
8	11	—						
9	10	5						
10	9	5	N.E. by N. ½ N.	Nil.	N.W. by W.	,,	16° W.	
11	9	5						
12	9	—	,,	,,	,,	,,		
1	9	5	N.N.E.	¼	N.W.	6	14° W.	
2	10	—						
3	10	5						
4	10	—	,,	,,	,,	,,		
5	9	5	N.E.	½	N.N.W.		19° W.	
6	8	5						
7	7	—	E.N.E.	¾	North.	7	12° W.	
8	5	—						
9	4	—	East.	1	N.N.E.	7	4° W.	A current set the ship during the 24 hours, correct magnetic N.N.W. 26 miles.
10	4	—						
11	4	5	E. ½ S.	1¼	,,		3° W.	
12	4	5	,,	,,		,,	,,	

Latitude 47° 11′ S. Longitude 49° 43′ E. Course and distance made good, N. 24°½ E., 197 miles.

No. 3.

.	K.	T.	Course.	Lee Way.	Wind.	Force.	Deviation.	Remarks.
1	5	5	South	1	W.S.W.	4	3° W.	Noon. A point of land in latitude 55° 21′ N., and longitude 179° 42′ W., bore by compass N. by W., distant 17 miles, ship's head being South.
2	5	—						
3	4	5						
4	4	5	”	½	W. by S.	3	”	
5	4	5						
6	4	—	”	Nil.	West.	3	”	Variation, 17° E.
7	4	5						
8	5	—				4		
9	5	5		Nil.	W. by N.			
10	6	5	”					
11	7	—						
12	7	5	”	”	W.N.W.	5		
1	8	—	S.W.	½	W.N.W.	6	8° E.	
2	9	—			N.W.	”		
3	9	—						
4	9	—	”	¾	”	”		
5	8	8						
6	8	2						
7	7	—						
8	6	—	”	¼	”	”		
9	5	5						A current set the ship correct magnetic South 14 miles during the last 24 hours.
10	5	5						
11	5	—						
12	5	—	”	”	”	”		

Latitude 53° 10′ 30″ N. Longitude 177° 37′ 0″ E. Course and distance made good, S. 36° W., 160 miles.

No. 4.

H.	K.	T.	Course.	Lee Way.	Wind.	Force.	Deviation.	Remarks.
1	4	5	West.	1	N.N.W.	3	19° W.	Noon. A point of land in latitude 49° 20′ N., and longitude 5° 20′ W., bore by standard compass S.S.W., distant 9 miles, ship's head being West. Variation 24° W.
2	4	5						
3	5	—						
4	5	—	W. by N.	$\frac{3}{4}$	N. by W.	,,	16° W.	
5	5	5						
6	5	5						
7	6	—	W. by N. $\frac{1}{2}$ N.	$\frac{3}{4}$	N. $\frac{1}{2}$ W.	,,	15° W.	
8	6	—						
9	6	—						
10	5	5	N.E. by E. $\frac{1}{2}$ E.	$\frac{1}{2}$	N. $\frac{1}{2}$ W.	,,	12° E.	
11	5	5						
12	6	—	,,	,,	,,	,,	,,	
1	5	8	E.N.E.	$\frac{3}{4}$	North.	3	13° E.	
2	5	5						
3	5	5						
4	5	2						
5	5	5	N.E. by E.	1	N. by W.	,,	10° E.	
6	6	1						
7	6	2						
8	6	2						A current set the ship during the 24 hours, correct magnetic North 30 miles.
9	6	—	N.E.	$\frac{3}{4}$	N.N.W.	,,	8° E.	
10	6	—						
11	5	5	West.	$\frac{1}{2}$	N.N.W.	,,	19° W.	
12	5	5	,,	,,	,,	,,	,,	

Latitude, 50° 0′ 30″ N. Longitude 5° 18′ 0″ W. Course and distance made good, N. 2° E., 41 miles.

LATITUDE BY MERIDIAN ALTITUDE.

SUN.

1. On February 22nd, 1895, in longitude 22° 30′ W., the observed meridian altitude of the sun's lower limb was 47° 20′ 10″, bearing south, index error +2′ 0″, height of eye 12 feet. Required the latitude.

Sun's declination at Greenwich apparent noon on 22nd .... 10° 10′ 27″·2 S.
Variation in one hour, decreasing, 54″·71.

Latitude, 32° 16′ 45″ N.

2. On March 21st, 1895, in longitude 100° E., the observed meridian altitude of the sun's lower limb was 79° 41′ 10″, bearing north, index error −1′ 40″, height of eye 20 feet. Required the latitude.

Sun's declination at Greenwich apparent noon on 21st .... 0° 15′ 6″·4 N.
Variation in one hour, increasing, 59″·23.

Latitude, 10° 0′ 24″ S.

3. On May 11th, 1895, in longitude 156° 14′ W., the observed meridian altitude of the sun's lower limb was 86° 20′ 40″, bearing north, index error +1′ 10″, height of eye 17 feet. Required the latitude.

Sun's declination at Greenwich apparent noon on 11th ... . 17° 53′ 18″·5 N.
Variation in one hour, increasing, 38″·5.

Latitude, 14° 33′ 37″ N.

4. On December 12th, 1895, in longitude 60° 2′ E., the observed meridian altitude of the sun's lower limb was

68° 12′ 40″, bearing south, index error −2′ 40″, height of eye 12 feet. Required the latitude.

Sun's declination at Greenwich apparent noon on 12th .... 23° 5′ 47″·6 S.
Variation in one hour, increasing, 11″·19.

Latitude, 1° 27′ 39″ S.

5. On June 1st, 1895, in longitude 35° W., the observed meridian altitude of the sun's lower limb was 85° 41′ 20″, bearing north, index error +0′ 50″, height of eye 19 feet. Required the latitude.

Sun's declination at Greenwich apparent noon on 1st .... 22° 3′ 54″·8 N.
Variation in one hour, increasing, 20″·39.

Latitude, 17° 58′ 22″ N.

6. On October 23rd, 1895, in longitude 18° E., the observed meridian altitude of the sun's lower limb was 75° 14′ 20″, bearing south, index error +4′ 10″, height of eye 25 feet. Required the latitude.

Sun's declination at Greenwich apparent noon on 23rd .... 11° 25′ 14″·2 S.
Variation in one hour, increasing, 52″·66.

Latitude, 3° 6′ 31″ N.

MOON.

7. On August 4th, 1895, at 11h. 28m. P.M., mean time at ship nearly, in longitude 117° 10′ E., the observed meridian altitude of the moon's lower limb was 42° 12′ 20″, bearing south, index error *nil*, height of eye 28 feet. Required the latitude.

Moon's declination at 3h., G.M.T., on the 4th .... 23° 13′ 53″·1 S.
Variation in 10 minutes, decreasing, 87″·78.

Semi-diameter	at noon, 14′ 52″·2,	at midnight, 14′ 50″·1.
Horizontal Parallax	„ 54′ 28″·9	„ 54′ 21″·0.

Latitude, 23° 49′ 29″ N.

8. On April 11th, 1895, at 1h. 18m. A.M., mean time at ship nearly, in longitude 47° 40′ W., the observed meridian altitude of the moon's lower limb was 74° 47′ 20″, bearing south, index error *nil*, height of eye 15 feet. Required the latitude.

Moon's declination at 16h., G.M.T. on the 10th......... 19° 17′ 37″·1 S.
Variation in 10 minutes, increasing, 127″·83.
Semi-diameter at midnight, 10th, 16′ 1″·3, at noon, 11th, 15′ 54″·9.
Horizontal Parallax ,, ,, 58′ 41″·9, ,, ,, 58′ 18″·4.

Latitude, 4° 38′ 28″ S.

9. On December 3rd, 1895, at 0h. 51m. A.M., mean time at ship nearly, in longitude 140° W., the observed meridian altitude of the moon's *upper limb* was 72° 25′ 20″, bearing south, index error +3′ 10″, height of eye 23 feet. Required the latitude.

Moon's declination at 22h., G.M.T. on 2nd .... 28° 8′ 18″·0 N.
Variation in 10 minutes, increasing, 4″·03.
Semi-diameter at midnight, 2nd, 15′ 40″·9, at noon, 3rd, 15′ 45″·1.
Horizontal Parallax ,, ,, 57′ 27″·3, ,, ,, 57′ 42″·8.

Latitude, 45° 43′ 16″ N.

STAR.

10. On September 26th, 1895, at about 10h. 30m. P.M., the observed meridian altitude of Markab, bearing south, was 65° 12′ 20″, index error +1′ 40″, height of eye 28 feet, Required the latitude.

Declination of Markab .... 14° 38′ 49″ N.

Latitude, 39° 30′ 28″ N.

11. On February 21st, 1895, at about 8·40 P.M., the observed meridian altitude of Sirius, bearing north, was

79° 21′ 50″, index error +2′ 10″, height of eye 30 feet. Required the latitude.

Declination of Sirius .... 16° 34′ 26″ S.

Latitude, 27° 16′ 1″ S.

12. On July 15th, 1895, at about 3·20 A.M., the observed meridian altitude of Fomalhaut, bearing south, was 65° 21′ 10″, index error, +1′ 20″, height of eye 16 feet. Required the latitude.

Declination of Fomalhaut .... 30° 10′ 24″ S.

Latitude, 5° 28′ 32″ S.

LATITUDE BY ALTITUDE OF POLARIS.

1. On February 7th, 1895, at about 9h. 30m. P.M., in longitude 42° W., the observed altitude of Polaris was 48° 12′ 30″, index error −1′ 10″, height of eye 14 feet. Required the latitude.

Right ascension mean sun at Greenwich mean noon on 7th .... 21h. 9m. 25·09s.

Latitude, 47° 54′ 47″ N.

2. On January 10th, 1895, at 2h. A.M., in longitude 170° W., the observed altitude of Polaris was 35° 24′ 40″, index error +2′ 35″, height of eye 20 feet. Required the latitude.

Right ascension mean sun at Greenwich mean noon on 10th .... 19h. 19m. 1·48s.

Latitude, 35° 59′ 13″ N.

3. On June 18th, 1895, at midnight, in longitude 144° E., the observed altitude of Polaris was 55° 22′ 10″, index

error +1′ 50″, height of eye 29 feet. Required the latitude.

Right ascension mean sun at Greenwich mean noon on 18th .... 5h. 45m. 53·87s.

Latitude, 55° 49′ 3″ N.

4. On September 30th, 1895, at 4 A.M., in longitude 27° W., the observed altitude of Polaris was 18° 41′ 30″, index error −3′ 10″, height of eye 25 feet. Required the latitude.

Right ascension mean sun at Greenwich mean noon on 29th .... 12h. 31m. 59·15s.

Latitude, 17° 41′ 2″ N.

5. On December 20th, 1895, at 5 P.M., in longitude 80° E., the observed altitude of Polaris was 52° 13′ 40″, index error +1′ 25″, height of eye 21 feet. Required the latitude.

Right ascension mean sun at Greenwich mean noon on 19th .... 17h. 51m. 20·23s.

Latitude, 51° 10′ 15″ N.

6. On April 9th, 1895, at 10 P.M., in longitude 130° W., the observed altitude of Polaris was 31° 49′ 10″, index error − 2′ 30″, height of eye 17 feet. Required the latitude.

Right ascension mean sun at Greenwich mean noon on 9th .... 1h. 9m. 54·9s.

Latitude, 32° 44′ 59″ N.

Latitude by Reduction to the Meridian.

1. On July 10th, 1895, at about 0h. 30m. P.M., in latitude D.R. 29° 30′ S, and longitude 114° E., when a watch showed 1h. 10m. 50s., whose error on apparent time at ship was found to be 0h. 44m. 50s. fast, the ship's run since being N. 65° E. (true) 31 miles, the observed altitude of the sun's lower limb near the meridian was 37° 42′ 50″, index error *nil*, height of the eye 14 feet. Required the latitude.

Sun's declination at Greenwich apparent noon on the 10th .... 22° 15′ 10′·5 N.
Variation in one hour, decreasing, 18″·93.

Latitude, 29° 22′ 14″ S.

2. On January 28th, 1895, at about 11h. 35m. A.M., in latitude D.R. 47° 10′ N., and longitude 45° 20′ W., when a chronometer showed 1h. 10m. 5s., whose error on G.M.T. was 1h. 39m. 46s. slow, the observed altitude of the sun's lower limb near the meridian was 24° 17′ 10″, index error *nil*, height of the eye 21 feet. Required the latitude.

Sun's declination at Greenwich mean noon on 28th ...... 18° 11′ 8″ S.
Variation in one hour, decreasing, 39″·64.
Equation of time to be subtracted from mean time, at Greenwich mean noon on 28th.................................. 13m. 9·91s.
Variation in one hour, increasing, ·469s.

Latitude, 47° 9′ 36″ N.

3. On March 1st, 1895, at about 0h. 30m. P.M., in latitude D.R. 39° 50′ S., and longitude 112° 40′ E., when a chronometer showed 5h. 1m. 40s., whose error on G.M.T. was 0h. 12m. 5s. slow, the observed altitude of the sun's

lower limb near the meridian was 56° 44′ 40″, height of the eye 24 feet. Required the latitude.

Sun's declination at Greenwich mean noon on the 1st ... 7° 34′ 2″·3 S.
Variation in one hour, decreasing, 57″·01.
Equation of time to be subtracted from mean time, at Greenwich mean noon on the 1st .... 12m. 31·65s.
Variation in one hour, decreasing, ·493s.

Latitude, 39° 57′ 41″ S.

4. On October 17th, 1895, at about 11h. 45m. A.M., in latitude D.R. 55° 18′ N., and longitude 145° 30′ W., when a chronometer showed 1h. 27m. 59s., whose error on G.M.T. was 7h. 45m. 47s. slow, the observed altitude of the sun's lower limb near the meridian was 25° 13′ 0″, height of the eye 32 feet. Required the latitude.

Sun's declination at Greenwich mean noon on 17th ... 9° 16′ 10″·5 S.
Variation in one hour, increasing, 54″·97.
Equation of time to be added to mean time, at Greenwich mean noon on 17th .... 14m. 34·06s.
Variation in one hour, increasing, ·498s.

Latitude, 55° 10′ 8″ N.

5. On December 2nd, 1895, at about 0h. 40m. P.M., in latitude D.R. 47° 15′ N., and longitude 147° 10′ W., when the time by a watch was 0h. 47m. 10s., whose error on apparent time at ship was found to be 6m. 50s. fast, after which the ship ran N. 10° W. (true) 35 miles; the observed altitude of the sun's lower limb near the meridian was 19° 47′ 30″, index error *nil*, height of the eye 16 feet. Required the latitude.

Sun's declination at Greenwich apparent noon on 2nd... 21° 58′ 37″·3 S.
Variation in one hour, increasing, 22″·24.

Latitude, 47° 25′ 29″ N.

6. On September 17th, 1895, at about 11h. 35m. A.M.,

in latitude D.R. 42° 31′ S., and longitude 91° E., when a chronometer showed 10h. 27m. 15s., whose error on G.M.T. was 5h. 0m. 29s. fast, the observed altitude of the sun's lower limb near the meridian was 44° 40′ 40″; I.E. + 1′ 50″, height of eye 10ft. Required the latitude.

Sun's declination at Greenwich mean noon on 17th ... 2° 15′ 18″·5 N.
Variation in one hour, decreasing, 58″·07.
Equation of time to be added to mean time, at Greenwich mean noon on 17th .... 5m. 30·24s.
Variation in one hour, increasing, ·881s.

Latitude, 42° 24′ 24″ S.

Longitude by Chronometer.

SUN.

1. On November 20th, 1895, at about 8.30 A.M., in latitude 20° 12′ N. and longitude (D.R.) 156° 5′ E., the observed altitude of the sun's lower limb was 24° 47′ 50″, index error + 2′ 0″, height of the eye 20 feet, when a chronometer showed 11h. 1m. 16s., which was fast of G.M.T. on September 30th 1h. 7m. 49s., and on October 31st it was fast of G.M.T. 1h. 9m. 22s. at mean noon at Greenwich. Required the longitude.

Sun's declination at mean noon Greenwich on November 19th .... 19° 29′ 11″·3 S.
Variation in one hour, increasing, 34″·91.
Equation of time at mean noon Greenwich on November 19th to be subtracted from apparent time............ 14m. 29·39s.
Variation in one hour, decreasing, ·571s.

Longitude, 156° 10′ 15″ E.

2. On March 17th, 1895, at about 4.30 P.M., in latitude 35° 20′ N. and longitude (D.R.) 156° 10′ W., the

observed altitude of the sun's lower limb was 17° 10′ 30″, index error −1′ 30″, height of the eye 26 feet, when the time by chronometer was 14h. 0m. 10s. on March 17th, whose error on G.M.T. was 1h. 4m. 43s. slow. Required the longitude.

Sun's declination at mean noon Greenwich on March 17th .... 1° 19′ 52″ S.
Variation in one hour, decreasing, 59″·26.
Equation of time at mean noon Greenwich on March 17th, to be added to apparent time .... 8m. 29·16s.
Variation in one hour, decreasing, ·73s.

Longitude, 156° 19′ 15″ W.

3. On January 15th, 1895, at about 3.30 P.M., in latitude 11° 15′ S., and longitude (D.R.) 106° 10′ W., when the time by chronometer was 10h. 27m. 36s., whose error on G.M.T. was 17m. 18s. slow, the observed altitude of the sun's lower limb was 38° 21′ 40″, index error + 3′ 20″, height of the eye 28 feet. Required the longitude.

Sun's declination at mean noon Greenwich on January 15th 21° 7′ 14″·5 S.
Variation in one hour, decreasing, 27″·69.
Equation of time at mean noon Greenwich on January 15th to be added to apparent time .... 9m. 37·49s.
Variation in one hour, increasing, ·877s.

Longitude, 105° 59′ 45″ W.

4. On April 6, 1895, at about 8 a.m., in latitude 40° 10′ N., and longitude (D.R.) 40° 20′ W., when the time by a chronometer was 21h. 41m. 27s. on April the 5th, whose error on G.M.T. on January 1st at mean noon Greenwich was 1h. 10m. 12s. slow, and its daily rate was 4 seconds gaining, the observed altitude of the sun's lower limb was 27° 6′ 0″, index error *nil*, height of the eye 12 feet. Required the longitude.

Sun's declination at mean noon Greenwich on April 6th 6° 27′ 53″·1 N.
Variation in one hour, increasing, 56″·62.
Equation of time at mean noon Greenwich on April 6th to be added to apparent time .... 2m. 28s.
Variation in one hour, decreasing, ·725s.

Longitude, 40° 9′ 30″ W.

5. On June 6th, 1895, at about 2 P.M., in latitude 53° 19′ S. and longitude (D.R.) 63° 20′ E., when the time by chronometer was 22h. 49m. 20s. on June 5th, whose error on G.M.T. was 59m. 12s. fast, the observed altitude of the sun's lower limb was 9° 10′ 20″, index error + 1′ 20″, height of the eye 22 feet. Required the longitude.

Sun's declination at mean noon Greenwich on June 6th 22° 39′ 50″·4 N.
Variation in one hour, increasing, 15″·5.
Equation of time at mean noon Greenwich on June 6th to be subtracted from apparent time .... 1m. 37·41s.
Variation in one hour, decreasing, ·445s.

Longitude, 63° 31′ 15″ E.

6. On June 14th, 1895, at about 3 P.M., in latitude 35° 20′ S., and longitude (D.R.) 100° 30′ E., when the time by a chronometer was 20h. 20m. 45s. on June 13th, whose error on G.M.T. was 2m. 0s. fast, the observed altitude of the sun's lower limb was 17° 23′ 10″, index error *nil*, height of the eye 22 feet. Required the longitude.

Sun's declination at Greenwich mean noon on June 14th 23° 16′ 36″·6 N.
Variation in one hour, increasing, 7″·43.
Equation of time at Greenwich mean noon on June 14th, to be subtracted from apparent time........................... 0m. 3·48s.
Variation in one hour, decreasing, ·525s.

Longitude, 100° 20′ E.

MOON.

7. On June 1st, 1895, at about 11h. 10m. P.M., in latitude 12° 40′ N., and longitude (D.R.) 175° 16′ E., the

observed altitude of the moon's lower limb (west of meridian) was 29° 13′ 50″, index error +1′ 40″, height of the eye 18 feet, when the time by a chronometer was 23h. 0m. 8s. on May 31st, whose error on G.M.T. was 18m. 30s. slow. Required the longitude.

Sidereal time at Greenwich mean noon on May 31st 4h. 34m. 55·82s.
Moon's declination at 23h. G.M.T. on May 31st......... 1° 27′ 40″·1 N.
Variation in 10 minutes, decreasing, 166″·88.
Moon's right ascension at 23h. G.M.T. on May 31st 11h. 40m. 30·17s
Variation in 10 minutes, increasing, 20·605s.
Moon's semi-diameter at midnight on May 31st .... 16′ 8″·1.
„ „ at noon on June 1st .... 16′ 6″·1.
„ horizontal parallax at midnight on May 31st .... 59′ 6″·9.
„ „ at noon on June 1st .... 58′ 59″·6.

Longitude, 175° 10′ E.

8. On October 30th, 1895, at about 1h. 10m. A.M., Apt. T. S., in latitude 48° 20′ S., and longitude (D.R.) 151° 45′ W., when the time by a chronometer was 22h. 48m. 25s. on October 29th, whose error on G.M.T. on June 1st at Greenwich mean noon was 0h. 24m. 14s. slow, and on August 1st 0h. 21m. 11s. slow, the observed altitude of the moon's lower limb was 21° 4′ 0″, index error +2′ 10″, height of the eye 23 feet. Required the longitude.

Sidereal time at Greenwich mean noon on October 29th.. 14h. 30m. 15·79s.
Moon's declination at 23h. G.M.T. on October 29th...... 2° 0′ 48″·7 N.
Variation in 10 minutes, increasing, 141″·83.
Moon's right ascension at 23h. G.M.T. on October 29th 0h. 1m. 21·84s.
Variation in 10 minutes, increasing, 17·583s.
Moon's semi-diameter at midnight on October 29th ... 14′ 48″·6.
„ „ at noon on October 30th ... 14′ 50″·3.
„ horizontal parallax at midnight on October 29th ... 54′ 15″·6.
„ „ at noon on October 30th ... 54′ 21″·9.

Longitude, 151° 40′ 30″ W.

STAR.

9. On February 17th, 1895, at about 1 A.M., in latitude 10° 12′ S., and longitude (D.R.) 7° 20′ E., when the time by a chronometer was 15h. 12m. 15s. on February 16th, whose error on G.M.T. was 2h. 26m. 25s. fast, the observed altitude of Sirius (west of meridian) was 26° 23′ 10″, index error −1′ 15″, height of the eye 19 feet. Required the longitude.

Sidereal time at Greenwich mean noon on the 16th	21h. 44m. 54·09s.
Right ascension of Sirius on February 20th	6h. 40m. 32·54s.
Declination of Sirius on February 20th	16° 34′ 25″·6 S.

Longitude, 7° 19′ 15″ E.

10. On April 23rd, 1895, at about 5 A.M., in latitude 42° 20′ N., and longitude (D.R.) 18° 17′ W., when the time by a chronometer was 20h. 19m. 15s. on April 22nd, whose error on G.M.T. was 1h. 58m. 55s. fast, the observed altitude of Markab (east of meridian) was 34° 11′ 40″, index error *nil*, height of the eye 29 feet. Required the longitude.

Sidereal time at Greenwich mean noon on the 22nd	2h. 1m. 10·11s.
Right ascension of Markab on April 21st	22h. 59m. 31·8s.
Declination of Markab on April 21st	14° 38′ 18″·6 N.

Longitude, 18° 10′ 45″ W.

AMPLITUDES.

1. On March 28th, 1895, at 5h. 50m. A.M., apparent time, in latitude 47° 50′ N., and longitude 10° 12′ W., the

sun when rising bore by standard compass S.E. by E., the variation being 24° W. Required the deviation.

Sun's declination at Greenwich apparent noon on 27th 2° 36′ 45″·4 ~~S.~~ N
Variation in one hour, increasing, 58″·71.

Deviation, 14° 5′ W.

2. On December 5th, 1895, at about 7 P.M., apparent time, in latitude 32° 12′ S., and longitude 140° 40′ W., the sun when setting bore by standard compass W.S.W., the variation was 10° E. Required the compass correction and deviation.

Sun's declination at Greenwich apparent noon on 5th.. 22° 23′ 23″·6 S.
Variation in one hour, increasing, 19″·03.

Compass correction... 4° 21′ 30″ W.
Deviation .... 14° 21′ 30″ W.

3. On June 17th, 1895, at about 3 A.M., apparent time, in latitude 57° 20′ N., and longitude 45° 10′ W., the sun when rising bore by standard compass E. ½ S., the variation was 48° W. Required the compass correction and deviation.

Sun's declination at Greenwich apparent noon on 16th 23° 21′ 43″·9 N.
Variation in one hour, increasing, 5″·37.

Compass correction ... 52° 58′ 17″ W.
Deviation .... 4° 58′ 17″ W.

4. On December 2nd, 1895, at about 5h. 30m. P.M., apparent time, in latitude 17° 20′ N., and longitude 160° 12′ E., the sun when setting bore by standard compass W. by S., the variation was 12° E. Required the compass correction and deviation.

Sun's declination at Greenwich apparent noon on 1st 21° 49′ 30″·9 S.
Variation in one hour, increasing, 23‴29.

Compass correction... 11° 47′ 30″ W.
Deviation .... 23° 47′ 33″ W.

5. On October 17th, 1895, at about 5h. 30m. A.M., apparent time, in latitude 40° 21′ S., and longitude 11° 12′ W., the sun when rising bore by standard compass E. 44° S., the variation was 22° W. Required the compass correction and deviation.

Sun's declination at Greenwich apparent noon on 17th 9° 15′ 57″·2 S.
Variation in one hour, increasing, 54″·97.

Compass correction... 31° 55′ W.
Deviation .... 9° 55′ W.

6. On July 4th, 1895, at about 3h. 37m. P.M., apparent time, in latitude 54° 15′ S., and longitude 47° 30′ E., the sun when setting bore by standard compass W. 50° N., the variation was 37° W. Required the compass correction and deviation.

Sun's declination at Greenwich apparent noon on 4th... 22° 53′ 37″ N.
Variation in one hour, decreasing, 13″·07.

Compass correction... 8° 15′ W.
Deviation .... 28° 45′ E.

ALTITUDE AZIMUTHS.

SUN.

1. On May 4th, 1895, at about 7.12 A.M., apparent time, in latitude 49° 20′ N. and longitude 37° 15′ W., the observed altitude of the sun's lower limb was 23° 21′, index error + 1′ 40″, height of the eye 22 feet, when the sun bore by standard compass S. 45° E., the variation being 35° W. Required, the compass correction and deviation on the course the ship was steering.

Sun's declination at Greenwich apparent noon on 4th 15° 58′ 28″·6 N.
Variation in one hour, increasing, 43″·43.

Compass correction... 42° 15′ 15″ W.
Deviation .... 7° 15′ 15″ W.

2. On June 29th, 1895, at about 8.45 A.M., apparent time, in latitude 22° 10′ S., and longitude 147° 12′ E., the observed altitude of the sun's lower limb was 22° 51′ 20″, index error *nil*, height of the eye 17 feet, when the sun bore by standard compass N. 56° 20′ E., the variation being 10° E. Required, the compass correction and deviation on the course the ship was steering.

Sun's declination at Greenwich apparent noon on 28th.. 23° 17′ 45″·2 N.
Variation in one hour, decreasing, 7″·01.

Compass correction ... 5° 2′ 45″ W.
Deviation .... 15° 2′ 45″ W.

3. On March 10th, 1895, at about 3.10 P.M., apparent time, in latitude 35° 22′ N., and longitude 178° W., the observed altitude of the sun's lower limb was 25° 47′ 10″, index error + 1′ 40″, height of the eye 14 feet, when the sun bore by standard compass S. 40° 40′ W., the variation being 12° E. Required, the compass correction and deviation on the course the ship was steering.

Sun's declination at Greenwich apparent noon on 10th...... 4° 5′ 6″ S.
Variation in one hour, decreasing, 58″·74.

Compass correction ... 23° 24′ E.
Deviation .... 11° 24′ E.

4. On January 27th, at about 4h. 30m. P.M., apparent time, in latitude 7° 10′ S. and longitude 72° E., the observed altitude of the sun's lower limb was 27° 15′ 20″, index error *nil*, height of the eye 21 feet, when the sun bore by standard compass S. 85° W., the variation being 1° E. Required, the compass correction and deviation on the course the ship was steering.

Sun's declination at Greenwich apparent noon on January 27th .... 18° 26′ 40″·9 S.
Variation in one hour, decreasing, 38″·81.

Compass correction ... 12° 6′ W.
Deviation .... 13° 6′ W.

5. On September 20th, 1895, at about 3h. 50m. P.M., apparent time, in latitude 57° 20′ N. and longitude 170° W., the observed altitude of the sun's lower limb was 18° 22′ 50″, index error + 1′ 10″, height of the eye 15 feet, when the sun bore by standard compass S. 74° 20′ W., the variation being 19° E. Required, the compass correction and deviation on the course the ship was steering.

Sun's declination at Greenwich apparent noon on September 20th .... 1° 5′ 31″·6 N.
Variation in one hour, decreasing, 38″·35.

Compass correction ... 13° 58′ 45″ W.
Deviation .... 32° 58′ 45″ W.

6. On November 30th, 1895, at about 6 A.M., apparent time, in latitude 51° 10′ S. and longitude 54° 15′ E., the observed altitude of the sun's lower limb was 16° 20′ 0″, index error − 1′ 30″, height of the eye 27 feet, when the sun bore by standard compass S. 30° 40′ W., the variation being 38° E. Required, the compass correction and deviation on the course the ship was steering.

Rot

Sun's declination at Greenwich apparent noon on November 29th .... 21° 30′ 3″·2 S.
Variation in one hour, increasing, 25″·36.

Compass correction ... 45° 7′ E.
Deviation .... 7° 7′ E.

STAR.

7. On August 16th, 1895, at 10h. 5m. P.M., in latitude 44° 20′ N. and longitude 45° 15′ W., the observed altitude of Arcturus (west of meridian) was 10° 27′ 40″, index error *nil*, height of the eye 20 feet, and it bore by standard compass N. 25° W., the variation being 27° W. Required, the compass correction and deviation on the course the ship was steering.

Arcturus's declination on August 19th .... 19° 43′ 39″ N.

Compass correction... 47° 38′ W.
Deviation .... 20° 38′ W.

8. On December 19th, 1895, at 9 P.M., in latitude 8° 30′ N. and longitude 140° 30′ E., the observed altitude of Sirius (east of meridian) was 29° 15′ 40″, index error + 1′ 20″, height of the eye 10 feet, when it bore by standard compass S. 71° 40′ E., the variation being *nil*. Required, the compass correction and deviation.

Sirius's declination on December 17th .... 16° 34′ 18″ S.

Compass correction... 6° 7′ E.
Deviation .... 6° 7′ E.

TIME AZIMUTHS.

SUN.

1. On February 10th, 1895, A.M. at ship, in latitude 44° 40′ N., and longitude 32° 30′ W., when the time by a chronometer was 19h. 22m. 44s. on the 9th, whose error on G.M.T. was 10m. 55s. fast, the sun bore by standard

compass south, the variation was 29° W. Required the compass correction and deviation.

Sun's declination at Greenwich mean noon on 10th ... 14° 20′ 1″·4 S.
Variation in one hour, decreasing, 48″·78.
Equation of time at Greenwich mean noon on 10th, to be subtracted from mean time .... 14m. 25·97s.
Variation in one hour, increasing, ·023s.
True azimuth by Burdwood .... N. 135° 30′ E.

Compass correction ... 44° 30′ W.
Deviation .... 15° 30′ W.

2. On May 19th, 1895, A.M. at ship, in latitude 33° 20′ N., and longitude 120° W., when a watch showed 8h. 10m. 20s., whose error on apparent time at ship was 18m. 20s. fast, the sun bore by standard compass S. 80° E., the variation was 10° E. Required the compass correction and deviation.

Sun's declination at Greenwich apparent noon on 19th .... 11° 10′ 59″·1 N.
Variation in one hour, increasing, 51″·87.
True azimuth by Burdwood .... N. 87° E.

Compass correction... 13° W.
Deviation .... 23° W.

3. On August 14th, 1895, in latitude 48° S., and longitude 54° 15′ E., at about 3h. P.M. apparent time at ship, the sun bore by standard compass N. 5° W., the variation was 35° W. Required the compass correction and deviation.

Sun's declination at Greenwich apparent noon on 14th .... 14° 22′ 33″·4 N.
Variation in one hour, decreasing, 46″·29.
True azimuth by Burdwood .... S. 134° 30′ W.

Compass correction ... 40° 30′ W.
Deviation .... 5° 30′ W.

4. On July 10th, at 8h. 30m. A.M. apparent time at ship nearly, in latitude 35° N., and longitude 18° W., the sun bore by standard compass east, the variation was 22° W. Required the compass correction and deviation.

Sun's declination at Greenwich apparent noon on 10th .... 22° 15′ 10″·5 N.
Variation in one hour, decreasing, 18″·93.
True azimuth by Burdwood .... N. 91° E.

Compass correction ... 1° E.
Deviation .... 23° E.

5. On October 21st, 1895, at 4h. 30m. P.M. apparent time at ship nearly, in latitude 47° N., and longitude 140° W., the sun bore by standard compass S. 71° W., the variation was 12° E. Required the compass correction and deviation.

Sun's declination at Greenwich apparent noon on 21st... 10° 42′ 46″ S.
Variation in one hour, increasing, 53″·5.
True azimuth by Burdwood .... N. 114° W.

Compass correction ... 5° W.
Deviation .... 17° W.

6. On April 9th, 1895, at 9h. A.M. apparent time at ship nearly, in latitude 58° S., and longitude 137° E., the sun bore by standard compass N. 30° E., the variation was 1° W. Required the compass correction and deviation.

Sun's declination at Greenwich apparent noon on 8th ... 7° 12′ 59″ N.
Variation in one hour, increasing, 56″·03.
True azimuth by Burdwood .... S. 133° 30′ E.

Compass correction ... 16° 30′ E.
Deviation .... 17° 30′ E.

STAR.

7. On January 19th, 1895, at 8h. P.M. mean time at ship nearly, in latitude 50° N., and longitude 19° W., the star Procyon bore by standard compass S. 48° E., the variation was 25° W. Required the compass correction and deviation.

Right ascension mean sun at Greenwich mean noon on 19th	19h. 54m. 30·51s.
Procyon's right ascension	7h. 33m. 50s.
,, declination	5° 29′ 42″ N.
True azimuth by Burdwood	N. 114° 30′ E.

Compass correction	17° 30′ W.
Deviation	7° 30′ E.

8. On March 17th, 1895, at 4h. A.M. mean time at ship very nearly, in latitude 37° S., and longitude 82° E., the star Altair bore by standard compass N. 63° E., the variation was 15° W. Required the compass correction and deviation.

Right ascension mean sun at Greenwich mean noon on 16th	23h. 35m. 18s.
Altair's right ascension	19h. 45m. 40s.
,, declination	8° 35′ 11″ N.
True azimuth by Burdwood	S. 114° 30′ E.

Compass correction	2° 30′ E.
Deviation	17° 30′ E.

9. On May 1st, 1895, at midnight mean time at ship very nearly, in latitude 40° N., and longitude 140° E., the star Regulus bore by standard compass N. 75° 30′ W., the

variation was 2° E. Required the compass correction and deviation.

Right ascension mean sun at Greenwich mean noon on 1st	2h. 36m. 39s.
Regulus right ascension	10h. 2m. 48s.
,, declination	12° 28′ 45″ N.
True azimuth by Burdwood	N. 93° 30′ E.

Compass correction	18° W.
Deviation	20° W.

Double Chronometer.

1. On April 22nd, 1895, at about 10h. 12m. P.M., in latitude D.R. 50° 7′ N., and longitude (D.R.) 47° 12′ W., when a chronometer showed 12h. 10m. 12s., on April 22nd, whose error on G.M.T. was 1h. 9m. 29s. slow, the observed altitude of Arcturus (east of meridian) was 51° 39′ 50″, and the observed altitude of Castor (west of meridian) was 35° 40′ 50″, I.E. *nil*, height of the eye 17 feet. Required the ship's position.

Sidereal time at Greenwich mean noon on 22nd	2h. 1m. 10·11s.
Right ascension of Arcturus	14h. 10m. 55s.
Declinination ,,	19° 43′ 27″ N.
Right ascension of Castor	7h. 27m. 55s.
Declination ,,	32° 7′ 17″ N.

Longitude by Arcturus	47° 39′ 30″ W.
Line of position ,,	N. 41° E. & S. 41° W.
Longitude by Castor	47° 27′ 45″ W.
Line of position ,,	N. 9½ E. & S. 9½ W.

True position of ship	Latitude	50° 17′ 15″ N.
	Longitude	47° 26′ 0″ W.

2. On November 24th, 1895, at about 3.20 A.M., in latitude D.R. 47° 20′ N., and longitude D.R. 2° 20′ W.,

when the G.M.T. by chronometer was 15h. 56m. 10s. on the 23rd, the observed altitude of Aldebaran (west of meridian) was 43° 32′ 10″, and that of Regulus (east of meridian) was 41° 2′ 30″, I.E. *nil*, height of eye 21 feet. Required the ship's position.

Sidereal time at Greenwich mean noon on 23rd ...	16h. 8m. 49·7s.
Right ascension of Aldebaran	4h. 29m. 58s.
Declination "	16° 18′ 11″ N.
Right ascension of Regulus	10h. 2m. 50s.
Declination "	12° 28′ 34″ N.
Longitude by Aldebaran..............	12° 6′ 45″ W.
" " Regulus	12° 34′ 0″ W.
Line of position by Aldebaran	N. 27½° W. & S. 27½° E.
" " by Regulus	N. 32° E. & S. 32° W.

True position of ship...	Latitude...	47° 36′ 18″ N.
	Longitude.	12° 19′ 10″ W.

3. On January 11th, 1895, at about 12h. 10m. A.M., in latitude D.R. 15° 30′ N., and longitude D.R. 154° 15′ E., when a chronometer showed 1h. 50m. 40s., on January 10th, whose error on G.M.T. was 2m. 2s. slow, the observed altitude of Sirius (west of meridian) was 55° 57′ 30″, and that of Regulus (east of meridian) was 52° 52′ 30″, I.E. *nil*, height of the eye 24 feet. Required the ship's position.

Sidereal time at Greenwich mean noon on 10th ...	19h. 19m. 1·48s.
Right ascension of Sirius	6h. 40m. 33s.
Declination "	16° 34′ 18″ S.
Right ascension of Regulus	10h. 2m. 48s.
Declination "	12° 28′ 46″ N.
Longitude by Sirius....................	153° 58′ 15″ E.
" " Regulus.................	154° 25′ 0″ E.

True position of ship by	Latitude...	15° 20′ 20″ N.
Blackburne's tables...	Longitude.	154° 25′ 0″

4. On July 18, 1895, at about 7 A.M., in latitude D.R. 50° 15′ N., and longitude D.R. 9° 30′ W., when the time by a chronometer was 19h. 30m. 50s., on the 17th, whose error on G.M.T. was 15m. 7s. slow, the observed altitude of the sun's lower limb was 25° 35′ 40″, and that of the moon's lower limb 61° 18′ 0″ (east of meridian), index error *nil*, height of the eye 22 feet. Required the ship's position.

Sidereal time at Greenwich mean noon on 17th	7h. 40m. 14·07s.
Sun's declination at „ „ on 18th	21° 2′ 37″·2 N.
Variation in one hour, decreasing, 26″·35.	
Equation of time at Greenwich mean noon on 18th to be added to apparent time	5m. 57·85s.
Variation in one hour, increasing, ·198s.	
Moon's declination at 19h. G.M.T. on 17th	25° 39′ 17″·3 N.
Variation in 10 minutes increasing, 73″·83.	
Moon's right ascension at 19h. G.M.T. on 17th	3h. 58m. 16·35s.
Variation in 10 minutes increasing, 24·262s.	
Moon's semi-diameter at midnight	15′ 41″
Horizontal parallax „	57′ 27″·5
Moon's semi-diameter at noon	15′ 48″·5
Horizontal parallax „	57′ 55″

True position...	Latitude...	50° 30′ N.
	Longitude.	9° 40′ W.

5. On November 21st, 1895, at about 2 P.M., in latitude D.R. 51° 15′ N. and longitude D.R. 7° 15′ E., when the G.M.T. by chronometer was 1h. 16m. 50s. on the 21st, the observed altitude of the sun's lower limb was 14° 9′ 30″, and that of the moon's lower limb was 8° 12′ 10″ (east of meridian), index error *nil*, height of the eye 18 feet. Required the ship's position.

Sidereal time at Greenwich mean noon on 21st	16h. 0m. 56·59s.
Sun's declination at Greenwich mean noon on 21st...	19° 56′ 23″·7 S.
Variation in one hour, increasing, 33″·12.	

Equation of time at Greenwich mean noon on 21st to be subtracted from apparent time	14m. 0·36s.
Variation in one hour, decreasing, ·638s.	
Moon's declination at 1h. G.M.T. on the 21st	23° 42′ 18″·4 S.
Variation in 10 minutes, decreasing, 84″·48.	
Moon's right ascension at 1h. G.M.T. on the 21st	20h. 5m. 6·6s.
Variation in 10 minutes, increasing, 21·78s.	
Moon's semidiameter at noon	15′ 9″·7
Horizontal parallax ,,	55′ 32″·9
Moon's semidiameter at midnight	15′ 14″·8
Horizontal parallax ,,	55′ 14″·8

True position... Latitude... 51° 5′ 15″ N.
Longitude. 7° 5′ E.

6. On March 2nd, 1895, at about 9 A.M., in latitude D.R. 47° 49′ N., longitude D.R. 48° 30′ W., when the G.M.T. by chronometer was 0h. 26m. 22s. on the 2nd, the observed altitude of the sun's lower limb was 22° 23′ 0″, and that of the moon's lower limb 5° 3′ 50″, index error *nil*, height of the eye 29 feet. Required the ship's position.

Sidereal time at Greenwich mean noon on 2nd	22h. 40m. 5·86s.
Sun's declination at Greenwich mean noon on 2nd	7° 11′ 10″·8 S.
Variation in one hour, decreasing, 57″·26.	
Equation of time at Greenwich mean noon on 2nd to be added to apparent time	12m. 19·57s.
Variation in one hour, decreasing, ·514s.	
Moon's declination at noon G.M.T. on 2nd	21° 3′ 55″·5 N.
Variation in 10 minutes, increasing, 109″·61.	
Moon's right ascension at noon G.M.T. on 2nd	2h. 52m. 36·74s.
Variation in 10 minutes, increasing, 21·567s.	
Moon's semidiameter at noon	15′ 24″·4
Horizontal parallax ,,	56′ 26″·6
Moon's semidiameter at midnight	15′ 29″·9
Horizontal parallax ,,	56′ 46″·9

True position... Latitude... 47° 38′ 37″ N.
Longitude. 48° 11′ W.

7. On June 2nd, 1895, at about 5.20 A.M., in latitude D.R. 51° 22′ N. and longitude D.R. 121° 9′ E., when the G.M.T. by chronometer was 9h. 15m. 38s. on the 1st, the observed altitude of the sun's lower limb was 11° 30′ 30″, height of the eye 10 feet. The ship then ran W.S.W. (true) 28′ till at about 10.30 A.M., when the G.M.T. by chronometer was 14h. 25m. 23s. on the 1st, and the observed altitude of the sun's lower limb was 55° 48′ 10″. Required the ship's position at the time of the second observation.

Sun's declination at Greenwich mean noon on the 1st .. 22° 3′ 55″·6 N.
Variation in one hour, increasing, 19″·42.
Equation of time at Greenwich mean noon on the 1st to be subtracted from apparent time .... 2m. 26·67s.
Variation in one hour, decreasing, ·373s.

True position... { Latitude... 51° 19′ 12″ N.
Longitude. 120° 33′ 42″ E.

8. On February 11th, 1895, at about 6h. 30m. A.M., in latitude D.R. 27° 32′ N. and longitude D.R. 155° 22′ E., when the G.M.T. by chronometer was 7h. 53m. 44s. on the 10th, the observed altitude of the moon's lower limb was 17° 6′ 0″ (west of meridian), height of the eye 15 feet. The ship then ran N. 77° E. (true) 35′, till about 10h. 30m. a.m., when the G.M.T. by chronometer was 12h. 21m. 6s. on the 10th, the observed altitude of the sun's lower limb was 42° 48′ 30″. Required the ship's position at the time of the second observation.

Sidereal time at Greenwich mean noon on the 10th... 21h. 21m. 14·76s.
Sun's declination at Greenwich mean noon on the 10th .... 14° 20′ 1″·4 S.
Variation in one hour, decreasing, 48″·78.
Equation of time at Greenwich mean noon on the 10th to be added to apparent time..................... 14m. 25·97s.
Variation in one hour, increasing, ·023s.

Moon's declination at 7h. G.M.T. on the 10th	10° 12′ 22″·8 N.
Variation in 10 minutes, decreasing, 171″·72.	
Moon's right ascension at 7h. G.M.T. on the 10th	10h. 36m. 58·26s.
Variation in 10 minutes, increasing, 22·932s.	
Moon's semidiameter at noon	16′ 44″·3
Horizontal parallax ,,	61′ 19″·5
Moon's semidiameter at midnight	16′ 41″·0
Horizontal parallax ,,	61′ 7″·4

True position...	Latitude...	27° 26′ 24″ N.
	Longitude.	155° 49′ 0″ E.

9. On April 6th, 1895, at about 3h. 25m. P.M., in latitude D.R. 48° 38′ N., and longitude D.R. 34° 24′ W., the observed altitude of the sun's lower limb was 29° 51′ 20″, when the G.M.T. by chronometer was 5h. 43m. 10s. on the 6th. The ship then ran N. 78° W. (true) 39′, till at about 7h. 15m. P.M., when the G.M.T. by chronometer was 9h. 31m. 52s. on the 6th, and the observed altitude of Sirius (west of meridian) was 21° 39′ 40″, height of the eye 26 feet, index error *nil*. Required the ship's position at the time of the second observation.

Sidereal time at Greenwich mean noon on 6th	0h. 58m. 5·24s.
Sun's declination at Greenwich mean noon on 6th	6° 27′ 53″·1 N.
Variation in one hour, increasing, 56″·62.	
Equation of time at Greenwich mean noon on 6th, to be added to apparent time	2m. 28s.
Variation in one hour, decreasing, ·725s.	
Right ascension of Sirius on April 11th	6h. 40m. 31·68s.
Declination of Sirius on April 11th	16° 34′ 27″·5 S.

True position...	Latitude...	48° 58′ 0″ N.
	Longitude.	35° 27′ 35″ W.

10. On August 14th, at about 3h. 0m. P.M., in latitude D.R. 35° 18′ S., and longitude D.R. 27° 34′ E., when G.M.T. by chronometer was 1h. 11m. 32s. on the 14th, the

observed altitude of the sun's lower limb was 25° 2′ 10″. The ship then ran S. 37° E. (true) 42′, when at about 7h. 30m. P.M., when the G.M.T. by chronometer was 5h. 38m. 48s. on the 14th, the observed altitude of Altair (east of meridian) was 31° 21′ 40″, height of the eye 20 feet, index error *nil*. Required the ship's position at the time of the second observation.

Sidereal time at Greenwich mean noon on 14th ...... 9h. 30m. 37·66s.
Sun's declination at Greenwich mean noon on 14th 14° 22′ 36″·9
Variation in one hour, decreasing, 46″·29.
Equation of time at Greenwich mean noon on 14th, to be added to apparent time .... 4m. 31·66s.
Variation in one hour, decreasing, ·46s.
Right ascension of Altair on August 19th ...... 19h. 45m. 42·87s.
Declination of Altair on August 19th .... 8° 35′ 34″·3 N.

True position... { Latitude... 35° 43′ 24″ S.
{ Longitude. 27° 52′ 50″ E.

11. On May 1st, 1895, at about 10h. 30m. A.M., in latitude D.R. 47° 15′ S., and longitude D.R. 98° 31′ E., when the G.M.T. by chronometer was 15h. 54m. 18s. on April 30th, the observed altitude of the sun's lower limb was 24° 15′ 30″. The ship then ran N. 80° E. (true) 45′ till at about 2h. 30m. P.M., when the G.M.T. was 19h. 49m. 57s. on April 30th, the observed altitude of the sun's lower limb was 19° 3′ 50″, index error *nil*, height of the eye 18 feet. Required the position of the ship at the time of second observation.

Sun's declination at Greenwich mean noon on April 30th .... 14° 46′ 57″ N.
Variation in one hour, increasing, 45″·99
Equation of time at Greenwich mean noon on April 30th to be subtracted from apparent time...... 2m. 52·59s.

True position... { Latitude... 47° 17′ 46″ S.
{ Longitude. 99° 14′ 46″ E.

12. On October 10th, 1895, at about 0h. 50m. P.M., in latitude D.R. 49° 40′ N., and longitude D.R. 10° 1′ W., when the G.M.T. by chronometer was 1h. 18m. 51s. on the 10th, the observed altitude of the sun's lower limb was 32° 18′ 0″. The ship then ran West (true) 34′ till about 4h. P.M. when the G.M.T. by chronometer was 4h. 31m. 21s. on the 10th, and the observed altitude of the sun's lower limb 13° 13″ 30″, index error *nil*, height of the eye 30 feet. Required the ship's position at the time of the second observation.

Sun's declination at Greenwich mean noon on the 10th .... 6° 39′ 22″·4 S.
Variation in one hour, increasing, 56″·88
Equation of time at Greenwich mean noon on the 10th to be subtracted from apparent time .... 12m. 56·52s.
Variation in one hour, increasing, ·657s.

True position... { Latitude... 49° 47′ 0″ N.
Longitude. 11° 2′ 0″ W.

GE

LOG OF THE STEAM YACHT "DEVIATION
THURS

H.	K.	T.	Course.	Lee Way	Wind.		De
					Direction.	Force.	
1					North	1 to 2	
2			At		,,		
3			anchor		,,		
4			in the		N.N.W.	2	
5			Downs.		,,		
6					,,		
7			Steering as necessary.		,,		
8			,,		N.W.	3	
9	10	—	S.W. ¾ W.		,,		1
10	1	2	,,				
	8	8	S.W. by W. ½ W.		,,		1
11	10	—	,,		,,		
12	10	—	,,		,,	2	

Latitude D.R.
Latitude bearing ... 50° 43′ N.

Longitude D.R.
Longitude beari

1	2	5	S.W. by W. ½ W.		N.W	1 to 2	1
	7	5	W. by S. ¼ S.				2
2	10	—					
3	10	—					
4	10	—	,,			2 to 3	
5	10	—					
6	9	8			,,	4	
7	9	8					
8	9	6	,,			5	
9	9	6					
10	9	8					
11	9	8				4	
12	10	—	,,				

PLATE A.

ERAL EXAMPLE.

ON A VOYAGE FROM THE DOWNS TO MILFORD HAVEN,
AY, AUGUST 15TH, 1895.

iation. | REMARKS.

A.M.—Swung ship for correction of compass. Deal Pier Head bearing correct magnetic W.N.W.

Ship's Head by Standard Compass.	Bearing Deal Pier Head.	Deviation.	Ship's Head by Standard Compass.	Bearing Deal Pier Head.	Deviation.
North	... N. 80° ½ W. ...	13° E.	South	... N. 50° ½ W. ...	17° W.
N.E.	... N. 63° ½ W. ...	4° W.	S.W.	... N. 75° ½ W. ...	8° E.
East	... N. 49° ½ W. ...	18° W.	West	... S. 89° ½ W. ...	23° E.
S.E.	... N. 44° ½ W. ...	23° W.	N.W.	... S. 89° ½ W. ...	23° E.

7.0. Weighed anchor in the Downs. Shaped course to pass about three miles from the South Foreland.* 8.0. South Foreland bore N.W. ¾ N. (N.N.W. magnetic), distant 3¼ miles. Shaped course S. 53° ½ W. (magnetic S. 66½ W.). Stream running N.E. about 1½ miles per hour. 9.45. Dungeness Lower Lighthouse bore west. Put over the patent log. 10.5. Dungeness Lower Lighthouse bore N.W. ¾ W. Patent log showed 3½ miles. Having doubled the bow angle, our distance from Dungeness Lower Lighthouse is 3½ miles, it being nearly slack water no allowance need be made for the stream. Altered course to S. 62° W. (magnetic S. 78° W. Noon. Royal Sovereign Light Vessel bore W. ¼ S. (magnetic N. 77° W.), distant three miles.

° E.

° E.

* When passing the South Foreland on a S. 27° W. course the compass bearing of the two lights in one was N. 79° W., the correct magnetic bearing being N. 81½ W., giving the deviation on this course as 2°½ W.

gs ... 0° 33′ E.

Course and distance made good,
Current course and distance

° E.
° E.

P.M. 12.15. Royal Sovereign Light Vessel N.N.W., distant two miles Altered course to S. 76° W. (N. 85° W. magnetic).

1.0. Beachy Head abeam, distant five miles.

5.26. H.W. at Dover. Stream slack.

,,

6.10. Sighted Dunnose Point bearing W. by N. ¾ N. Put over patent log.

6.55. Dunnose Point bore N.W. by N. and patent log showed 7½ miles; allowing 1¼ miles for west going stream, our distance from Dunnose Point is 8¾ miles.

7.20. Sun, when setting, bore by compass W. 20° N., giving deviation by amplitude as 20° E.

,,

7.30. St. Catherine's Point abeam.

9.30. Sighted Anvil Point Light abeam.

10.10. Sighted Portland High Light bearing N.W. by W. ¼ W. Anvil Poin Light bearing N. ¼ E.

10.20. Portland Lights in one bore N. 54° W., giving the deviation as 20° E. Midnight.

,,

LOG OF THE STEAM YACHT "DEVIATION"

FRIDA

H.	K.	T.	Course.	Lee Way	Wind.		Devia
					Direction.	Force.	
1	10	—	W. by S¼ S.		N.N.W.	2	20°
2	10	—					
3	9	8			North	,,	
4	10	—	,,				
5	10	—					
6	10	—			,,	,,	
7	9	8					
8	9	8	,,		,,	1	
9	10	—					
10	10	2			Calm	0	
11	10	—					
12	10	2	W.¼ N.		,,	,,	24°

Lat. D.R. 49° 46′ N. Long. D.R. ... 5° 23′ W. Course and d
Lat. Obsn. 49° 53′ N. Long. bearings 5° 22′ W. Current cours

H.	K.	T.	Course.	Lee Way	Direction.	Force.	Devia
1	10	—	W.¼ N.		Calm	0	24°
2	10	2					
3	10	2	N. by W.				16°
4	10	—			S.E.	1	
5	9	8					
6	5	—			South	,,	
	5	—	N.N.E.¾ E.				3°
7	10	2					
8	10	2			Calm	0	
9	1	5					
	8	5	N.E. by E.				8°
10	7	—					
11	3	—					
12	3	—	,,		,,	,,	,,

ON A VOYAGE FROM THE DOWNS TO MILFORD HAVEN,
, AUGUST 16TH, 1895.

ion.	REMARKS.
E. E.	A.M.—12.45. Lost sight of Portland High Light bearing N.N.E.½ E. 3.0. The observed altitude of Polaris was 51° 29′ 20″; index error —1′ 10″; height of eye, 15ft.; which gives the latitude as 50° 10′ N. 3.15. Sighted the Start Light bearing W. by N.½ N. Put over patent log. 4.10. Start Light bore N.W.¼ N., and patent log showed 9 miles. Allowing 1 mile for east-going stream the Start Point is 8 miles distant. 4.40. Start Point abeam. 4.50. Sun rose, bearing E. 24° N., which gives the deviation on a W. by S.¼ S. southerly course as 20° E. 6.6. H.W. at Dover. 7.40. Lat. D.R. 50° 0′ N., Long. D.R. 4° 16′ W., when the chronometer showed 7h. 8m. 13s., whose error on G.M.T. was 1h. 10m. 12s. slow, the observed altitude of the sun's L.L. was 29° 3′ 50″. This gives longitude 4° 16′ W. The sun bore at the same time, by standard compass, S. 77° E., giving the deviation as 20′ E. 10.15. Sighted the Lizard bearing W.N.W. Put over patent log. 11.0. Lizard bore N.N.W¾ W. Patent log showed 7½ miles, which, as there is no stream, is our distance from the Lizard. Altered course to W.¼ N. (magnetic N. 63° W.). Deviation 24° E. 11.30. Lizard Point abeam. Noon. Observed. mer. alt. of sun's L.L. was 53° 40′ 30″., giving latitude 49° 53′ N., and longitude by bearing (N.E.¼ E.) of Lizard 5° 22′ W.

stance made good. S. 78° W., 234 miles. True bearing and distance of nearest danger.
e and distance Wolf Rock, N. 75° W., 18 miles.

E. E. . .	P.M. 12.30. L.W. at Dover. 1.0. Sighted the Wolf Lighthouse one point on the starboard bow. 1.50. Wolf Lighthouse North, distant two miles. 2.0. Altered course to N. by W. (magnetic N. 5° E.). Deviation 16° E. 2.30. Longships Lighthouse bore N.E. by E.¼ E.; Wolf Lighthouse bore S.E.½ E. 5.30. Altered course to N.N.E.¾ E. (magnetic N.E. by N.). Deviation 3° E. The chronometer showed 4h. 38m. 2s. Error on G.M.T. slow 1h. 10m. 12s. The observed altitude of the sun's lower limb was 16° 45′ 20″. The latitude by D.R. from the position at 2.30 being 50° 27′ N. The ship then ran N. 17° E. (true) 27 miles when at 8.10 the chronometer showed 7h. 27m. 27s., and the observed altitude of Antares (W. of Mer.) was 10° 31′.* The true position of the ship from these two observations was latitude 50° 53′ N., and longitude 5° 54′ W. 8.10. Altered course to N.E. by E. (magnetic N. 48° E.). Deviation 8° W. 9.30. Weather getting thick; slowed down to 3 knots. H.W. at Liverpool at 7.18 p.m. The stream will set to the eastward till after midnight at the rate of about 1½ knots per hour. Midnight.

* True bearing is found by Johnson's Bearings of Bright Stars or by Altitude Azimuth.

LOG OF THE STEAM YACHT "DEVIATION" C
SATURDA

H.	K.	T.	Course.	Lee Way	Wind.		Deviati
					Direction.	Force.	
1	3	—	N.E. by E.		West	1	8° W
2	3	—					
3	3	—			N.W.	"	
4	4	5					
5	6	—			North	1 to 2	
6	6	—					
7	7	—	Steering as necessary.		"	3	"
8							
9			At				
10			anchor				
11			off				
12			Milford.				

Lat. D.R. Long. D.R. Course an
Lat. Obsn. Long. Chro. Current c

1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							

PLATE C.

N A VOYAGE FROM THE DOWNS TO MILFORD HAVEN,
, AUGUST 17TH, 1895.

n.	REMARKS.
	A.M. 12.30. Sounded in 44 fathoms, which, corrected for height of tide, is 41¾ fathoms. 3.30. Fog lifting a little, proceeded at 6 knots. 4.30. Sounded in 37 fathoms, which, corrected for height of tide, is 35 fathoms. 5.30. ,, 33 ,, ,, ,, 31½ ,, 6. 0. ,, 31 ,, ,, ,, 29½ ,, Slowed down to 3 knots. 6.15. Sounded in 30 fathoms, which, corrected for height of tide, is 29 ,, 6.30. Fog cleared. Sighted { St. Ann's Head, bearing N.E. by E. ½ E. (N. 56° E., magnetic). Linney Head, bearing E. by S. ½ S. (S. 81° E., magnetic). Went ahead full speed, steering as necessary for the anchorage off Milford. 7.30. Came to off Milford in 7 fms., veered 30 fms. chain. Noon.

distance made good. urse and distance.	True bearing and distance of nearest danger.

CUTTER YACHT “MAGNETIC” ON A VOY
SUNDA

H.	K.	T.	Course.	Lee Way	Wind. Direction.	Wind. Force.	Weath
1					S.W.	1	
2							
3			At Anchor in				
4			Waterford		„	1 to 2	
5			Harbour.				
6					„	2	
7			Beating out.				
8					„	3	
9	5	2	S. ¼ W.	—	S.W. by W.		
10	5	4					
11	5	5					
12	5	5	„	—	„	„	

Lat. D.R. 51° 44′ N. Long. D.R. 6° 40′ W. Course and
Lat. Obsn. 51° 42′ 30″ N. Long. Chro. Current co

H.	K.	T.	Course.	Lee Way	Wind. Direction.	Wind. Force.	Weath
1	5	5	South.		S.W. ½ W.	3	Fine and
2	5	4	S. ½ E.				Smooth
3	5	5	S. by E.		S.W. ½ S.	„	
4	5	6	S.S.E.		S.S.W. ½ W.	„	„
5	5	4	S. by E. ½ E.		S.W. by S.	„	
6	5	—	S. ½ E.		S.W.	„	„
7	4	8	S. by E.		S.W. ½ S.	„	
8	4	8	„		„	„	
9	4	8	S.S.E.		S.S.W. ½ W.	„	„
10	5	—	S.S.E. ½ E.		S.S.W.		
11	5	2	S.E. by S.		S. by W. ½ W.	„	„
12	5	—	S.E.		S. ½ W.	„	„

PLATE D.

GE FROM WATERFORD TO COWES, *viâ* CHERBOURG,
Y, MAY 26TH, 1895.

r.	REMARKS.
	A.M. 5.30. Weighed and beat out of the harbour. 7.0. Hook Point bore N.E., distant 2 knots. 8.0. Saltees Light Vessel bore East; Hook Point bore North. Took departure. Put over patent log. Latitude 52° 0′ 30″ N.; Longitude 6° 51′ W. 9.30. Lost sight of Saltees Light Vessel bearing N.E. ½ N. Stream running to the eastward about 1 knot per hour (page 124 Admiralty Tide Tables). Noon. Patent log showed 21¼ knots. Observed mer. alt. of sun's lower limb 59° 15′ 30″; Index Error—2′ 0″; height of eye 11ft. Now slack water (page 124 Admiralty Tide Tables). Re-set patent log.

distance made good S. 22° E. 20′
urse and distance.

True bearing and distance of nearest danger
Seven Stones Light Vessel S. 13° E., 101 miles.

lear. sea.	P.M.—4.0. Stream has set the ship S.E. about 3 knots since noon. The chronometer showed 5h. 17m. 26s., whose error on G.M.T. was 0h. 54m. 23s. fast when the observed altitude of the sun's lower limb was 34° 37′ 10″, Index Error and Dip. as above, giving the longitude as 6° 14′ 15″ W., with the latitude D.R. 51° 23′ 20″ N., the true bearing of the sun being at the same time (*vide* Burdwood) S. 80° W., the ship must be somewhere on the line of position S. 10° E., and N. 10° W. through the place of the ship as given by latitude D.R. 51° 24′ N. at right angles to the true bearing of the sun. 8.0. When the chronometer showed 9h. 18m. 13s., the observed altitude of Arcturus was 51° 31′ 30″ (E. of Meridian). I.E. — 2′, height of eye 11ft., giving longitude 5° 57′ 45″ W., with latitude D.R. 51° 5′ 30″ N. The true bearing of Arcturus at the same time was S. 46° E. by Burdwood. The true place of the ship by projecting the result of these two observations on the chart is 51° 6′ N. and 5° 58′ W. 9.50. The observed meridian altitude of Arcturus was 58° 51′ 40″. I.E.—2′, height of eye 11ft., giving the latitude as 50° 57′ 43″ N.—Midnight. : The observed altitude of Polaris was 50° 5′ 0″, I.E.—2′, height of eye 11ft., giving the latitude as 50° 53′ N., and allowing for the vessel having been set to the westward by the tidal stream since 9.50 p.m. 2 knots, her position is latitude 50° 53′ N., longitude 5° 37′ W. Tacked ship.

CUTTER YACHT "MAGNETIC" ON A VOY

MONI

H.	K.	T.	Course.	Lee Way	Wind. Direction.	Wind. Force.	Weat
1	5	—	S.W.		South.	3	Fine an Sea a littl
2	5	—	,,				
3	4	8	S.W. by W.		S. by W.	,,	
4	4	7	,,		,,	,,	,,
5	4	8	S.W.		South.	,,	
6	5	—	,,			3 to 4	
7	5	5	,,				
8	5	6	,,		,,	3	
9	2 3	5 —	W.S.W.		,,		,,
10	6	—	,,			3 to 4	Westerl
11	6	—	,,				
12	6	—	,,		,,	,,	,,

Lat. D.R. 50° 4′ 42″ N. Long. D.R. 6° 21½′ W. Course
Lat. Obsn. Long. Chro. Curren

H.	K.	T.	Course.	Lee Way	Wind. Direction.	Wind. Force.	Weat
1	5	5	W.S.W.		South.	4	Fine an
2	5	5	S.W. ½ S.		S. ½ E.	,,	
3	5	5	,,				Clou
4	5	—	,,			3	
5	3	—	W.S.W.		S.S.W.	2	
6	6	—	E. by S. ½ S.		S.Wʸ.	4	Rain s
7	7	5					
8	7	5	,,		,,	5 to 6	,,
9	8	—					
10	8	—					
11	7	5					
12	7	5	,,		,,	,,	,,

AGE FROM WATERFORD TO COWES, *viâ* CHERBOURG,
DAY, MAY 27TH, 1895.

her.	REMARKS.
d clear. e lumpy.	A.M.—2.16. The observed meridian altitude of Vega, bearing south, was 78° 1′ 30″, I.E.—2′, height of eye 11ft., giving latitude as 50° 45′ N. 4.0. Tidal stream set the ship since midnight N.E. 5 miles. * 4.30. When the chronometer showed 5h. 49m. 1s., error of chronometer 54m. 23s. fast, the observed altitude of Altair was 46° 2′ 10″, I.E.—2′, height of eye 11ft., giving longitude with a D.R. latitude of 50° 39′ 30″ N. as 5° 54′ 0″ W. True bearing, Altair S. 23° W. (Burdwood). Line of position N. 67° W. and S. 67° E. The ship then ran S.W. 20·4 miles, the tidal stream having set her during the four hours N.E. 5 knots, when at 8.30. the same chronometer showed 9h. 50m. 11s., and the observed altitude of the sun's lower limb was 40° 34′ 15″, I.E.—2′, height of eye 11ft., giving longitude 5° 50′ 30″ W., the latitude D.R. being 50° 24′ 30″ N., and the sun's true bearing (Burdwood) being S. 72½° E., giving lines of position N. 17½° E. and S. 17½° W. The above double chronometer gives the true position of the ship as latitude 50 22′ N., longitude 5° 51½′ W. Altered course to W.S.W. Reset patent log.
y swell.	10.30. Sighted the Scilly Islands ahead. 12.0 Noon. Round Island Lighthouse bore S.¾ W. Tidal stream set the ship during the last four hours W.S.W. 5 knots.

* In working out the dead reckoning the run of the ship from 4.0 to 4 30 is evidently S.W. 2·4 knots.
" " " " 8.30 to 12.0 is W.S.W. 21 knots.

and distance made good S. 6° E. 99′. True bearing and distance of nearest danger.
t course and distance East 9·4′. Round Island S. 11° E. 4 miles.

l clear.	P.M.—1.0. Round Island Lighthouse bore E. by S. ¾ S. Bishop's Rock Lighthouse bore S. by W. Hauled close to the wind. From this time till 6 p.m. the tidal stream is setting North at the rate of from 1½ to 2 knots per hour.
dy.	2.50. Bishop's Rock Lighthouse abeam.
quall.	4.0. Bishop's Rock Lighthouse bore N.E. by E. When the chronometer showed 5h. 16m., error of chronometer 54m. 23s. fast, the observed altitude of the sun's lower limb was 35° 28′ 30″, I.E.—2′, height of eye 11ft., which, combined with the bearing, gives the true position of the ship as latitude 49° 43′ N., longitude 6° 37′ 30″ W.
	5.0. Tacked ship. Shaped course E. by S. ½ S. Reset patent log.
	7.10. St. Agnes Light abeam.
	8.0. St. Agnes Light N. by W.
	9.0. The tidal stream has set the ship since 5.0 about E. by S. ¾ knot per hour.
	11.30. Stream commencing to run to the westward.
	Midnight. Lizard Lights N.N.E. ¾ E. Patent log 51½ knots.

CUTTER YACHT "MAGNETIC" ON A VOY
TUESD

H.	K.	T.	Course.	Lee Way	Wind. Direction.	Wind. Force.	Weat
1	7	5	E. by S. ½ S.		S.W.	6	Occasion
2	7	5					squall
3	7	5					Fog
4	3	—	"		W.S.W.	3	
5	2	5				2	
6	2	5					Fo
7	2	5					
8	2	5	"		"	3	
9	2	5					Very t
10	2	5					
11	2	5					Baromete
12	2	5	"		"	"	"

Lat. D.R. 49° 47′ N. Long. D.R. 4° 5′ W. Course and d
Lat. Obsn. Long. Chro. Current cours

H.	K.	T.	Course.	Lee Way	Wind. Direction.	Wind. Force.	Weat
1	2	5	E. by S. ½ S.		S.W. by W.	4	Thi
2	3	—					
3	1	5	Hove to on port tack.		S.W.	5 to 6	"
4							Squa
5							
6					S.S.W.	7	Heavy s
7							
8			"				
9					S. by W.	8 to 9	"
10			Hove-to on starboard tack.		S.S.W.		
11					S.W.	8	
12					W.S.W.	7	"

PLATE F.

AGE FROM WATERFORD TO COWES, *viâ* CHERBOURG,

AY, MAY 28TH, 1895.

her.	REMARKS.
al rain s.	A.M.
g.	High water at Dover at 1h. 24m. a.m.
	12.45. Lizard Lights abeam.
	3.0. Patent log showed 74¼ miles. Weather becoming very thick, sounded fog horn three blasts every two minutes and shortened sail.
g.	5.0. Stream is estimated to have set the ship since midnight west about 3 knots.
	6.0. Weather cleared. Made sail.
	7.0. Came over thick. Shortened sail. Kept fog horn going.
hick.	11.30. The stream is estimated to have set the ship since 5 a.m. east about 4 knots.
r falling.	
.	Noon.

istance made good,
e and distance,

True bearing and distance of nearest danger.
Les Hanois, Guernsey, S. 70° E., 59 miles.

ck.	P.M.
ly.	2.30. Barometer falling fast and heavy squalls coming up. Hove to on port tack under double reefed mainsail, reefed staysail and shifted jibs.
ualls.	6.0. Lowered and stowed mainsail and set trysail. The stream is estimated to have set the ship west 9 knots since noon.
	9.0. Very heavy squalls.
	9.30. Wind veering and glass rising. Shifted to starboard tack. Estimated drift since 2.30 p.m., N.N.W. 12 knots.
	12.0. Midnight. The stream is estimated to have set the ship E.S.E. 12 knots since 6 p.m. Estimated drift since 9.30 p.m., S.E. 5 knots.

CUTTER YACHT "MAGNETIC" ON A VOY
WEDNE

H.	K.	T.	Course.	Lee Way	Wind. Direction.	Wind. Force.	Weat
1			Hove to on Starboard Tack.		West.	6	Clea
2					W.N.W.	5	Fi
3	8	—	E. ¾ N.		N.W.	,,	
4	8	—					
5	8	—			,,		
6	4	—	,,				
	4	—	E.N.E.				
7	8	—					
8	8	—			,,	,,	,,
9	8	—			,,		
10	8	—	East.				
11	2	—					
	6	—	S.S.E. ¼ E.			4	
12	8	—	Steering as necessary, rounding east end of breakwater.		,,	,,	,,

Lat. D.R.
Lat. Obs.

Long D.R.
Long. Chron.

Course and
Current c

H.	K.	T.	Course.	Lee Way	Wind. Direction.	Wind. Force.	Weat
1	4	5	As necessary.		N.W. by W.	5	
2			At anchor.				
3			Steering as necessary.				
4	7	8	N.E. ½ E.		,,	,,	
5	8	5					
6	9	—					
7	9	—					
8	9	—			,,	,,	
9	4	5					
	4	0	N. by W.				
10	7	—					
11	7	—	N.N.W.		W.N.W.	,,	
12	5	—	,,				
	2		W.S.W.		,,		

PLATE G.

GE FROM WATERFORD TO COWES, *viâ* CHERBOURG,
DAY, MAY 29th, 1895.

er.	REMARKS.
ng.	A.M.—Estimated drift of ship till 2 a.m. being S.S.E. 3 knots; tidal drift W. by S. 4 knots. 2.0. Chronometer showed 3h. 6m. 45s., whose error on G.M.T. was 54m. 53s. fast. The following simultaneous altitudes were observed: Markab 24° 39′ 0″ (E. of Mer.); Altair 45° 38′ 50″ (E. of Mer.); Arcturus 32° 23′ 0″ (W. of Mer.). No index error; height of eye 11ft., the estimated latitude being 49° 45′ N., and longitude 3° 56′ W. The true position of the ship from these observations is latitude 49° 31′ N., longitude 3° 37′ 30″ W. Shaped course E. $\frac{3}{4}$ N. Set mainsail. 5.30. Chronometer showed 6h. 35m. 21s. Observed altitude sun's lower limb 11° 44′ 40″. Error of chronometer 54m. 33s. fast, I.E. *nil*, height of eye 11ft. With an estimated latitude of 49° 39′ N., this gives the longitude as 3° 1′ 30″ W. Tidal stream till 8 a.m. estimated south $7\frac{1}{2}$ knots. Altered course to E.N.E. 8.0. Sighted the Casquets bearing S.E. by E. $\frac{1}{4}$ E. 8.35. Casquets bore S.E. by S. $\frac{1}{2}$ S. distant $4\frac{1}{2}$ knots. 9.0. Altered course to E. 10.15. Cape La Hogue Light House bearing S. 4° W. Altered course to S.S.E. $\frac{1}{4}$ E. Set squaresail. Stream setting east at the rate of 5 knots per hour since 9.0 a.m. Noon.

distance made good.

rse and distance.

True bearing and distance of nearest danger.

P.M.—12.40. Came to in Cherbourg Roads in 6 fms., veered 30 fms. cable.

2.30. Weighed anchor and stood out of the Roads. 2.45. Rounded the east end of breakwater.

3.0. Breakwater Light House bore S.S.W. distant 2 knots. Shaped course N.E. $\frac{1}{2}$ E. Stream will set to the westward at the rate of 3 knots till 9 p.m.

5.30. When the chronometer showed 6h. 27m. 1s., Error 54m. 33s. fast, the observed altitude of sun's lower limb was 21° 6′ 30″, I.E. 0, and at the same time the observed altitude of the moon's lower limb was 56° 54′ 10″ (W. of Mer.) I.E. 0; height of eye 11ft., which give the true position of the ship as latitude 50° 1′ 20″ N., longitude 1° 31′ 30″ W.

8.30. Sighted St. Catherine's Light bearing N.N.E. $\frac{1}{2}$ E. Altered course to North.

8.40. Hauled close to the wind. Stream setting to the S. Eastward about $2\frac{1}{2}$ knots per hour.

9.30. Sighted Needles Light bearing N. $\frac{1}{2}$ W. St. Catherine's Light bore N.E. by E. $\frac{3}{4}$ E.

11.40. Tacked.

Midnight. Needles Light N.E. $\frac{1}{2}$ N. St. Catherine's Light S.E. by E. $\frac{3}{4}$ E.

CUTTER YACHT "MAGNETIC" ON A VOYAG
THURSDA

H.	K.	T.	Course.	Lee Way	Wind. Direction.	Wind. Force.	Weather.
1	1 5	5 5	W.S.W. N.N.W.		W.N.W.	5	Fine.
2	9	—	As necessary.				
3	2	—	"		"	"	"
4							
5							
6			At				
7			Anchor				
8			in				
9			Cowes				
10			Roads.				
11							
12							

Lat. D.R. Long. D.R. Course and di
Lat. Obsn. Long Chro. Current cour

1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							

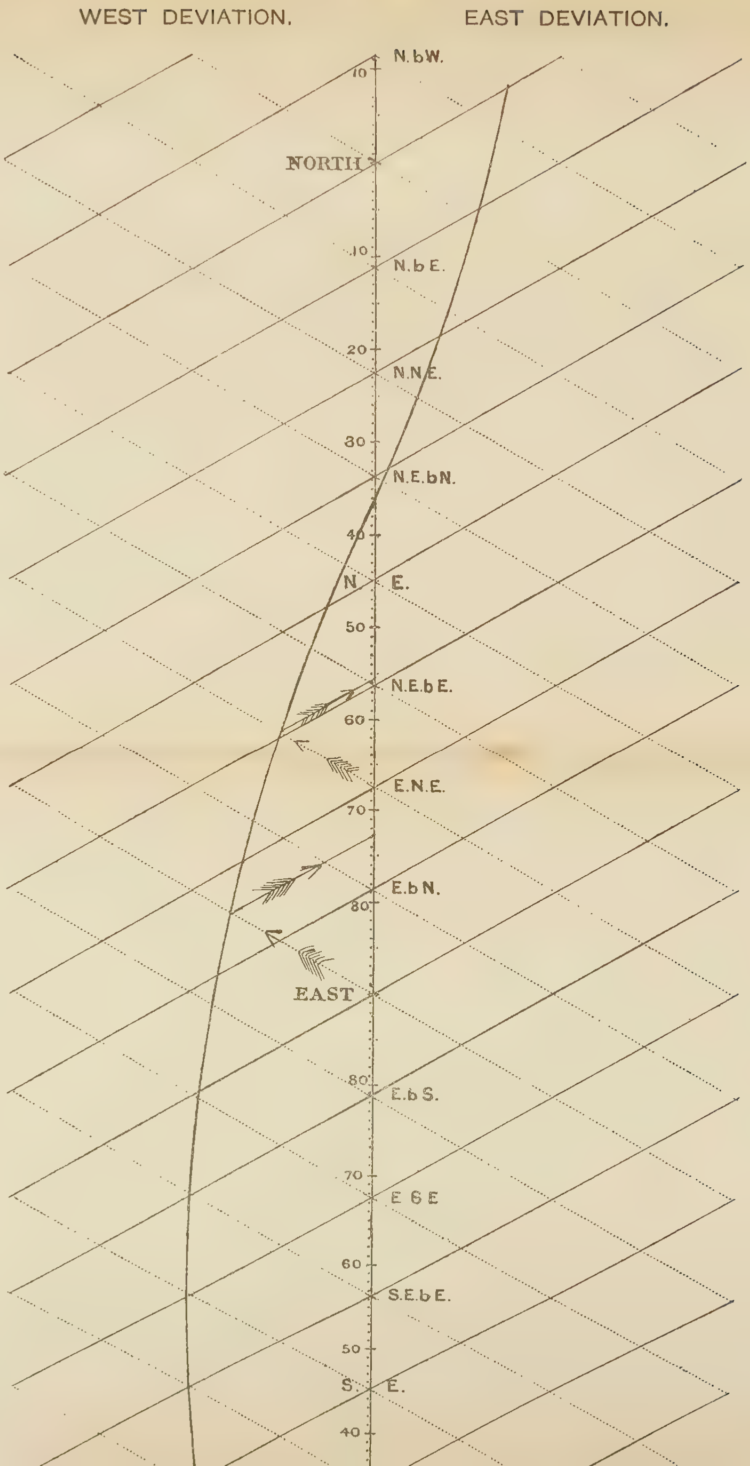
PLATE H.

FROM WATERFORD TO COWES, *viâ* CHERBOURG,
, MAY 30th, 1895.

	REMARKS.
	A.M. 12.10. Tacked ship. 1.0. Bore up, steering as necessary through the Needles Channel. 2.15. Shortened sail and came-to in Cowes Roads in 5 fms., veered 30 fms. cable.

stance made good. e and distance.	True bearing and distance of nearest danger.

PLATE I.


WEST DEVIATION.
EAST DEVIATION.
N.bW.
10
NORTH
10
N.b E.
20
N.N.E.
30
N.E.bN.
40
N
E.
50
N.E.b E.
60
E.N.E.
70
E.b N.
80
EAST
80
E.b S.
70
E 6 E
60
S.E.b E.
50
S.
E.
40

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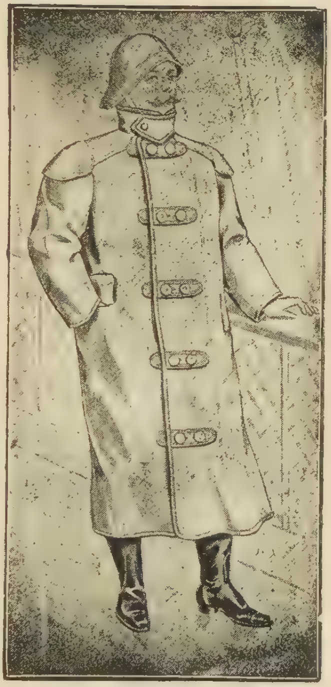

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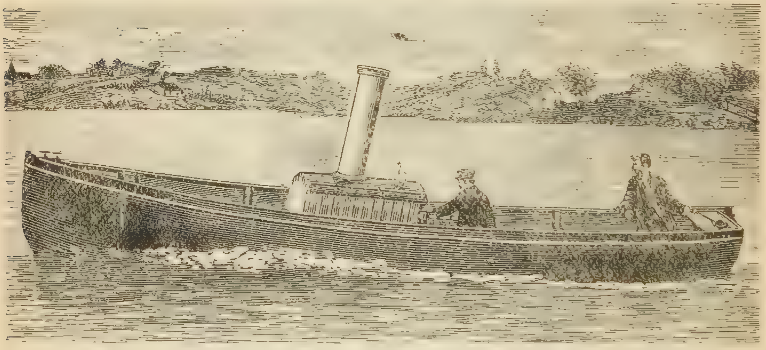

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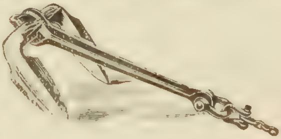

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